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The Relationship between Self-Regulation and Proportional Reasoning: The Mediating Role of Reflective Thinking towards Problem Solving

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Abstract

The present study examines the mediating impact of reflective thinking on problem-solving among seventh-grade students, focusing on the link between self-regulation and proportional reasoning. A relational research model was used to provide the quantitative research design. The prediction model proposed in this study was tested with 279 seventh-grade middle-school students. Data were collected using the 'Reflective thinking towards problem-solving scale', the 'Perceived self-regulation scale' and the 'Proportional reasoning skill test'. Correlation and mediation analyses were performed on the collected data. The results demonstrate that reflective thinking toward problem-solving is significantly related to self-regulation and proportional reasoning. The mediation analysis shows that seventh-grade students' reflective-thinking skills towards problem-solving partially mediate between their self-regulation levels and proportional-reasoning skills.

Keywords

Reflective thinking
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Introduction

Many researchers have emphasised the importance of self-regulation in the development of mathematical reasoning (arithmetic, algebraic, and proportional reasoning) (Kramarski, 2008; Pape, Bell, & Yetkin, 2003; Wolters & Pintrich, 1998). In particular, studies have shown that self-regulated learning improves mathematical success and mathematical reasoning in research carried out using learning methods based on self-regulation (Camahalan, 2006; Cleary & Kitsantas, 2017; Yaniawati, Kartasasmita, Kariadinata, & Sari, 2017). Mathematics word problem-solving (Holmes, Spence, Finn, Ingram, & Horton, 2017) and mathematical success (Cleary, Velardi, & Schnaidman, 2017) are frequently discussed in studies that examine the effect of self-regulation or self-regulated learning on mathematical reasoning. This is not the case for proportional reasoning, even though proportional reasoning plays a very important role in mathematics education. Proportional reasoning creates a link or a bridge between arithmetic reasoning and algebraic reasoning. It examines the relationship between different variables, rather than the relationship between concrete objects (Akkuş & Duatepe-Paksu, 2006). As the effects of self-regulation and reflective thinking on proportional reasoning have never investigated, this study has created a model to establish a link between self-regulation and proportional-reasoning ability. The model conceptualises the relation between self-regulation and proportional reasoning, mediated by reflective thinking about a problem. In doing so, it reveals that variables that influence the development

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of proportional-reasoning ability. Learning environments designed around these variables can contribute to developing proportional-reasoning ability.

Self-Regulation and Proportional Reasoning

One component of mathematical reasoning is proportional reasoning (Lesh, Post, & Behr, 1998). Proportional reasoning is defined as a cognitive process involving the representation of a state related to rate-proportion, symbolic representation, modelling, problem-ratio solving, and multiple comparisons (Özgün-Koca & Altay, 2009). According to Akkuş and Duatepe-Paksu (2006), when proportional reasoning requires the comprehension of multiplicative relations, its structure consists of the following: the discovery of unknown dimensions, quantitative comparisons, qualitative comparisons, and inverse-proportion algorithms. Modestou and Gagatsis (2010) have argued that proportional reasoning is a ternary model of higher-order analogical reasoning, requiring analogical reasoning, routine proportions, and metacognitive skills. The model proposed in the present study draws attention to the metacognitive dimension of proportional reasoning. The cognitive dimension includes awareness of proportional and non-proportional situations. Qualitative reasoning (making nonverbal comparisons) is related to the metacognitive dimension of proportional reasoning. As Modestou and Gagatsis (2010) have argued, proportional reasoning involves cognitive and metacognitive processes, with an emphasis on the need to control, monitor, and regulate one's own cognitive processes in proportional reasoning. Research has shown that self-regulation, which consists of three components: cognition, metacognition, and motivation (Zimmerman, 1990), is important for proportional reasoning.

Many studies link self-regulation to metacognition (Alpaslan, Yalvac, Loving, & Willson, 2016; Chen & Chiu, 2016). Tock and Moxley (2017) note that metacognition plays an important role in self-regulation. According to A. Arslan (2016), the cognitive dimension of self-regulation is related to metacognition, while the social dimension differs from metacognition. Self-regulation can be defined as organising one's own learning by determining self-learning goals, providing motivation, and using cognitive skills, such as planning, monitoring, and control (Arslan & Gelişli, 2015). Individuals with self-regulation skills can develop their skills with knowledge of their own learning (Zimmerman, 1990). Individuals with high-order self-regulatory skills can monitor and control themselves even more effectively (Arslan & Gelişli, 2015). Thus, prompts provided by self-regulatory components, such as planning, monitoring, control, and evaluation, foster the development of higher-order thinking skills (Karakelle, 2012), including critical thinking, reflective thinking, inquiry (Tock & Moxley, 2017), and evaluation (Banarjee & Kumar, 2014; Zimmerman, 1990). As self-questioning is a very important component of self-monitoring and control, the development of an individual's self-regulation skills strengthens the development of interrogation skills. As S. Arslan (2016) notes, the self-evaluation process, which is recognised as one of the stages of self-regulation, enables individuals to evaluate their own performances and recognise their inadequacies. Thus, developing self-regulatory skills may help individuals develop evaluation skills. The questioning and evaluation skills that we consider important for self-regulation constitute the basis of reflective thinking, alongside other components (Kızılkaya & Aşkar, 2009).

Self-Regulation and Reflective Thinking

Individual skills, such as planning, questioning, justifying, reasoning, and evaluating, require awareness – and thus provide a basis for both self-regulation and reflective thinking. Lee (2005) has described reflective thinking as a specific state of being aware of one's own learning. Halpern (2007) has argued that most reflective thinking in education is metacognitive reflective thinking. Reflective thinking is generally defined as being aware of what we can do, in addition to what we do and why, taking into account previously experienced phenomena Woolfolk-Hoy, 2015. Reflective thinking can help individuals develop interrogation, comparison, and reasoning skills (Tok, 2008). Individuals with reflective-thinking skills can interrogate their own learning processes (Schaaf, Baartman, Prins, Oosterbaan, & Schaap, 2013), planning (Kaplan, Doruk, & Öztürk, 2017), and evaluations (Kızılkaya & Aşkar, 2009; Mansvelder-Longayroux, Beijaard, & Verloop, 2007). Studies in the reflective-thinking

literature (Gelter, 2003; Dewey, 1933) focus on various components, including suggestion, planning, hypotheses, testing, evaluation, and justification). The common components discussed in these studies are questioning, reasoning, and evaluation (Kızılkaya & Aşkar, 2009; Schaaf et al., 2013). Interrogation involves searching for answers to questions that occur in an individual's own mind or come from outside. Reasoning is the act of thinking about one's own behaviour by considering the reasons for that behaviour. Evaluation is the process of looking back at one's own behaviour and determining whether it was right or wrong (Kızılkaya & Aşkar, 2009).

Reflective Thinking and Proportional Reasoning

According to Agustan, Juniati, and Siswono (2017) reflective thinking helps people solve problems by facilitating reasoning, especially in complex algebraic problems. Similarly, Van de Walle, Karp, and Bay-Williams (2013) have argued that making comparisons (i.e., comparing contrasting ratios) with fractions, which form the basis for proportional reasoning and a conceptual understanding of rate concepts, develops reflective-thinking skills. However, contrary views also exist in the literature. For example, Riebman and Overton (1977) found no significant difference in the proportional-reasoning ability of students who exhibited reflective-thinking skills and those who did not exhibit reflective-thinking (problem-solving) skills. Kaput and West (1994) found that students could perform automated processing of ratio-to-fraction information based on the fractions they learned in primary school. Thus, students can ignore some skills, such as questioning, reasoning, and evaluation. Shin, Jonassen, and McGee (2003) have shown that students ignore certain skills, such as planning, monitoring, and regulating, when confronted with problems they have previously encountered. In other words, students solve the problem of proportional reasoning automatically, without employing reflective-thinking skills.

Rationale for the Study

The inclusion of proportional reasoning in the curriculum of many countries clearly demonstrates the importance of teaching mathematics (National Council of Teachers of Mathematics [NCTM], 2000). According to NCTM (2000), proportional-reasoning skills derived from mathematical thinking must be developed to improve students' justification skills. The development of proportional reasoning has been described as a broad bridge, which starts with fractions, continues on to rational numbers, and extends as far as functions (Kaput & West, 1994). The Ministry of National Education (MoNE) has found that proportional reasoning has important applications in students' daily lives (MoNE, 2018). Studies have shown that features such as justification, questioning, evaluation, and metacognitive awareness are in the foreground during the development of proportional-reasoning ability (Akkuş & Duatepe-Paksu, 2006; Modestou & Gagatsis, 2010). However, to our knowledge, no study has yet compared the relationships that exist between proportional reasoning, self-regulation, and reflective thinking toward problem solving. In determining the variables that affect proportional-reasoning skills, this study is expected to contribute significantly to the literature on proportional reasoning. Another departure from previous research is the fact that this study considers the cognitive and affective domains together. Proportional reasoning is a cognitive skill, which previous studies have examined from a cognitive perspective (Arican, 2019; Ayan & Bostan, 2018; Howe, Nunes, & Bryant, 2011). However, it can be easier for students to acquire mathematical skills when the cognitive features of mathematics are supported by affective features (Öztürk, Akkan, & Kaplan, 2019). For this reason, it is important to identify the affective characteristics associated with proportional-reasoning ability. The present study therefore examines the mediating impact of reflective thinking on problem-solving among seventh-grade students, focusing on the link between self-regulation and proportional reasoning. In line with this purpose, Figure 1 presents the model tested in this study.

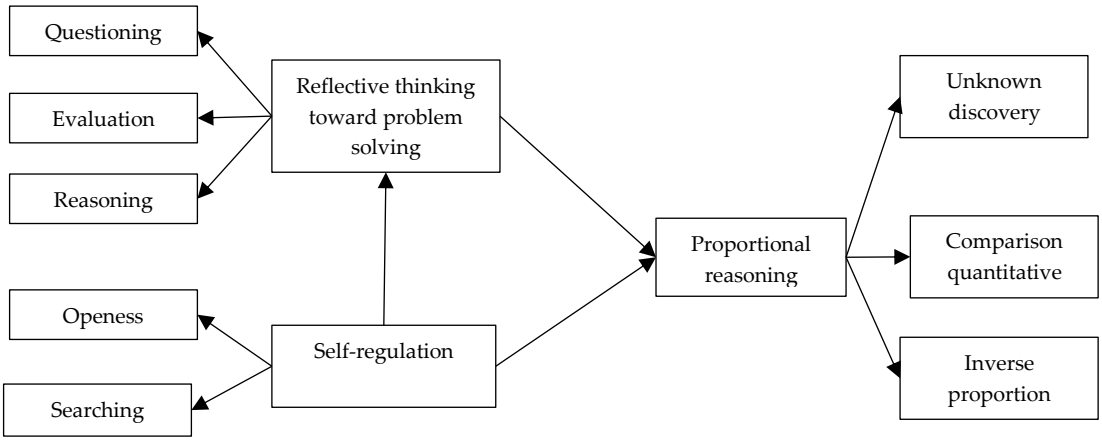

Figure 1. The Prediction Model Proposed in this Study

The prediction model in Figure 1 uses three sub-dimensions mentioned by Kızılkaya and Aşkar (2009) to define reflective thinking ability used for problem solving. They note that these three variables are commonly used sub-dimensions in studies that deal with reflective thinking for problem solving. For self-regulation, the openness and searching sub-dimensions discussed by Arslan and Gelişli (2015) have been used in the model. Arslan and Gelişli (2015) called the searching for the efforts of middle-school students to accept mistakes and the openess to make mistakes that they could accept (being open to criticism for not accepting mistakes). For this reason, openness and searching are discussed in this study as two sub-dimensions of self-regulation. With the literature in mind, the inverse proportion, quantitative comparison, and qualitative comparison sub-dimensions relate to proportional reasoning.

The prediction model was tested with the following hypotheses:

1. H_1: The self-regulation levels of seventh-grade students directly affect their reflective thinking toward problem solving.
2. H_2: The reflective-thinking skills that seventh-grade students use for problem-solving directly affect their proportional-reasoning skills.
3. The predictive effect of seventh-grade students' self-regulation of proportional reasoning is significant when the effect of reflective thinking on problem-solving is controlled.

Method

Research Model

A relational-research model, drawn from quantitative research designs, was used in this study. Relational research is often used to test the existence of correlations (Fraenkel, Wallen, & Hyun, 2012). McMillan and Schumacher (2014) divided relational research into bivariate correlations, prediction studies, and multiple correlations. The present study is a prediction study, carried out to determine the variables that describe proportional-reasoning ability.

Participants

Ratios and proportions are included in the sixth- and seventh-grade mathematics curriculum in Turkey. While sixth graders learn about ratios, seventh graders also learn about proportions (MoNE, 2018). For this reason, proportional reasoning, based on basic concepts of ratios and proportions, is fully covered in seventh-grade mathematics (Akkuş & Duatepe-Paksu, 2006). Since proportional-reasoning skills are positioned between arithmetic and algebraic reasoning (Akkuş & Duatepe-Paksu, 2006), the best learning grade for measuring this skill is Grade 7. As a result, most studies of proportional reasoning have been conducted at this grade level (Akkuş & Duatepe-Paksu, 2006; Çelik & Yetkin-Özdemir, 2011; Kaplan & Öztürk, 2012).

Two hundred seventy-nine (279) middle-school students in seventh-grade, studying in a city in eastern Turkey, participated voluntarily in the current study. A purposeful sampling method was used for the sampling. To carry out the study, official permission was first obtained from the Provincial Directorate of National Education. Next, interviews were held with school principals. Any school that did not approve the study was removed from the study sample. Student volunteers were asked to study and participants were selected. Research was carried out (apart from school classes) over two 40-minute periods of time. During the first period, students were asked to answer questions about the independent variables. After a break, they were asked to solve questions on a proportional-reasoning skills test during the second period. To ensure that the students focused on the questions, they were told that the students with the highest scores would receive a reward.

Instruments

Proportional-Reasoning Skills Test

The 'Proportional Reasoning Ability Test' developed by Akkuş and Duatepe-Paksu (2006) was used to determine the proportional-reasoning skills of seventh-grade students. After conducting an exploratory factor analysis (EFA), the researchers determined that the 15-question test had a two-factor structure. The first factor covered the discovery of unknown dimensions, quantitative comparisons, and inverse proportions. The second factor covered qualitative comparisons (comparisons without numerical values). The test rubric developed by the researchers was used in the evaluation. Three different grading keys were created to match the types of questions in the test. Questions related to unreported findings were scored between 0 and 3, and a 4-point scoring key was created. Questions related to quantitative comparisons, qualitative comparisons, and inverse-proportion algorithms were scored between 0 and 4; a 5-point scoring key was created. The Cronbach' s alpha reliability coefficient in the original form of the scale was reported as .86

The confirmatory factor analysis (CFA) conducted to examine the validity of the two-factor scale structure suggested that the two-factor structure was not appropriate. For this reason, to investigate the factor structure of the elements, an EFA was carried out, a CFA was performed, and the test-structure validity was examined.

Çokluk, Şekercioğlu, and Büyüköztürk (2014) noted two assumptions for conducting an EFA. These assumptions were appropriate for the Kaiser-Meyer-Olkin (KMO) sample size value and the Barlett Test of Sphericity. In the present study, we began by calculating the KMO value to test the sample size. The KMO values in this study were .90. According to Seçer (2013), a value of .70 or above indicates that the sample is sufficient. In this context, the calculated value showed that the sample size was appropriate. The results of the Barlett test also showed that the normality assumptions were met. Based on the assumptions, an EFA was carried out. From the exploratory factor analysis, a structure explaining 58.23% of the total variance, consisting of 16 questions with 3 factors, was obtained. These factors were appropriate for the types of questions ('quantitative comparison, qualitative comparison, inverse proportion, and discovery of unknown dimensions') that Akkuş and Duatepe-Paksu (2006) proposed. It was determined that the factor loadings of the items ranged from .43 to .85. The first factor in the scale accounted for 29.13% of the total variance. This factor included questions about 8 unknown items on the test. The second factor accounted for 17.05% of the total variance. Along with 4 qualitative comparison questions), the inverse proportionality algorithm was included within this factor. The third factor accounted for 12.35% of the scale and included 3 questions with a quantitative comparison. Table 1 presents the variance explained by the factor loadings of the scale items.

Table 1. Factor Loadings and Item-Total Correlations for the Items in the Proportional-Reasoning Test

	Factors			Item-total correlations
	Discovery unknown	**Inverse proportion**	**Quantitative comparison**	
Item 1	.657			.522
Item 2	.659			.490
Item 3	.773			.639
Item 4	.819			.760
Item 5	.845			.782
Item 6	.668			.624
Item 7	.732			.585
Item 8	.669			.561
Item 12		.685		.560
Item 13		.754		.660
Item 14		.757		.658
Item 15		.639		.479
Item 16		.563		.337
Item 9			.428	.442
Item 10			.737	.649
Item 11			.752	.616
Total variances	%29.3	%17.05	%12.35	
		Total Variances: %58.23		

As Table 1 shows, the factor loadings ranged from .66 to .85 for the discovery of unknown dimensions, from .57 to .76 for qualitative comparison and inverse proportion, and from 43 to .76 for quantitative comparison. According to Seçer (2013) factor-loading values must be .30 or above. In this context, the values were suitable for the item factor-loading values.

A CFA was carried out to determine the appropriate structure of the scale. Fit indices (X^2/df=2.66, RMSEA=.077, SRMR=.05, NNFI=.97, CFI=.95, IFI=.94, AGFI=.86) were calculated. These values indicated that the scale had acceptable fit indices. The researchers indicated that, although the test consisted of questions involving the discovery of unknown dimensions, quantitative comparisons, qualitative comparisons, and inverse-proportion algorithms, only one problem was related to inverse-proportion algorithms. The EFA and CFA showed that questions involving the discovery of unknown dimensions, inverse proportions, and quantitative comparisons constituted one factor. For this reason, the factors in this scale refer to the discovery of unknown dimensions, inverse proportion, and quantitative comparisons. Questions based on qualitative comparisons and inverse-proportion algorithms revealed that these areas constituted a single factor. An example of the first factor is as follows:

> Mr. Short has a friend named Mr. Tall. When Mr. Short is measured using a paperclip, he is found to be six paperclips tall. When Mr. Tall and Mr. Short are measured using a cufflink, Mr. Tall is 6 cufflinks tall and Mr. Short is 4 cufflinks tall. According to this, how many paper clips tall is Mr Tall?

The second factor concerns qualitative comparisons and inverse-proportion algorithms. The following is a qualitative-comparison question from the test: '*On a jogging track, Elif ran more laps in less time than Emel. Which one is faster? Please explain'.* This question involves an inverse-proportion algorithm: '*Nevzatcan and Nergis walk at the same speed around a track. Nevzatcan began walking before Nergis did. When Nevzatcan completes 9 rounds, Nergis completes 3 rounds. When Nergis completes 15 rounds, how many rounds does Nevzatcan complete? Please explain'.* Here is a quantitative-comparison question from the test: '*Flatbreads baked in a restaurant are always the same size. Seven girls who eat at this restaurant share 3 flatbreads equally, while 3 boys share 1 flatbread equally. Does 1 girl eat more flatbread than 1 boy in this restaurant? Please explain.'*

The reliability coefficient of the achievement test was calculated as .89. It was therefore above .65, as suggested by Field (2013). The collected data were therefore reliable.

Perceived Self-Regulation Scale

The 'Perceived Self-Regulation Scale', developed by Arslan and Gelişli (2015) to measure the self-regulation skills of middle-school students, was used to determine the participants' self-regulation skill levels. According to the researchers who developed the scale, 16 items, collected into two factors, explained 54.3% of the total variance via the EFA. These factors were called 'openness' and 'searching'. The five-point Likert scale ranged from 1 (never) to 5 (always). In the original form of the scale, the Cronbach' s alpha reliability coefficient was .90 for the whole scale, .84 for the openness dimension, and .82 for the searching dimension. The test questions included the following: '*I can do my work in accordance with the goals I have set.*' and '*If you fail, change the way you work'.*

As a result of the confirmatory factor analysis (CFA), conducted to determine the accuracy of the scale structure within this study, the compliance indices were calculated as ($X^2_{(103,\ N=279)}$=293.27, RMSEA=.071, SRMR=.06, NNFI=.96, CFI=.97, IFI=.97, AGFI=.85). These values show that the scale had acceptable fit indices. The reliability value of the complete scale was .91; according to Field (2013), a reliability value of .70 or above is sufficient. For this reason, the collected data are understood to be reliable.

Reflective-thinking-skills Scale for Problem Solving

The 'Reflective Thinking Skill Scale towards Problem Solving', developed by Kızılkaya & Aşkar (2009) is used to determine the extent to which students think reflectively when solving problems. The scale developed for seventh-grade students consists of 14 items and three factors. These factors are questioning, reasoning, and evaluation. A CFA was used to examine the scale's construct validity. The literature uses CFA as a sub-dimension of reflective thinking because questioning, reasoning, and evaluation are used intensively. In the present study, the scale was appropriate, given the content validity. In the original form of the scale, the Cronbach's alpha reliability coefficient was .83 for the scale, .71 for the reasoning dimension, .69 for the evaluation dimension, and .73 for the questioning dimension. Items from the text included the following: 'When solving a problem, I do what I do', 'After I solve the problem and find the result, I check the actions I make'.

The compliance indices were calculated as ($X^2_{(74,\ N=279)}$=162.81, RMSEA=.070, SRMR=.06, NNFI=.96, CFI=.97, IFI=.97, AGFI=.88), based on the CFA conducted to determine the accuracy of the scale structure in this study. These values show that the scale had an acceptable level of compliance. The reliability of the scale was .86. According to Field (2013), a reliability value of .70 or above is sufficient. For this reason, the collected data are understood to be reliable.

Data Analysis and Procedure

Inferential statistics were used in the data analysis. When estimating predictive statistics, descriptive statistics were used where needed. To analyse the data obtained in this study, predictive statistics, a form of quantitative data analysis, were used. Predictive statistics are often used when descriptive statistics cannot predict the results that predictive statistics could achieve (McMillan & Schumacher, 2014). As in this study, this method of analysis is used to determine the variables that predict proportional-reasoning skills.

SPSS 18 and Lisrel 8.80 software were used in the data analysis. First, normal distributions of data were examined. For a normal distribution, kurtosis and skewness values were considered, and a histogram chart was created. It determined that the kurtosis and skewness values were within the ±1 range for all variables. At the same time, the histogram chart shows that the data were near a normal distribution. By drawing a box-and-whisker diagram, 5 sets of data that broke at normality were

removed from the dataset. Normality was controlled by carrying out the same procedures again until it was clear that there was no problem. Afterwards, the Mahalonobis distance value, recommended for multivariate analyses (Seçer, 2013), was examined; the highest value was found to be 12.52. This provided a multivariate normality hypothesis for all scales.

The relations between variables were examined using a correlation analysis and all relations were found to be at a significant level. Subsequently, prediction models were created. Regression analyses are generally used for prediction models. However, a regression analysis provides no cause-and-effect relation. By contrast, structural equality modelling (SEM) can set up cause-and-effect relationships between variables and also specify the intermediary variables (Çelik & Yılmaz, 2016; Şimşek, 2007). For this reason, this study used a mediation analysis, a type of structural equation modelling.

Results

The Relationship between Proportional Reasoning Skills and the Predicting Variables

Information about the validity and reliability of the measuring instruments used in the study is presented in the data-analysis section, together with an explanation of the normality hypothesis. Table 2 presents descriptive statistical results obtained from measuring instruments and correlation values.

Table 2. Descriptive Statistical and Correlation-Analysis Results Related to Measuring Instruments

	Bivariate Correlation			**Descriptive statistics**			
	1	**2**	**3**	**M**	**SD**	**Skewness**	**Kurtosis**
1. Reflective thinking toward problem solving	1	-.57**	-.42**	32.29	9.85	.322	-.175
2. Perceived self-regulation	-	1	.38**	61.27	11.33	-.593	.479
3. Proportional reasoning	-	-	1	21.21	12.34	.311	.726

Table 2 shows a significant relationship between proportional-reasoning skills, reflective thinking toward problem solving, and self-regulation levels. The variable 'proportional reasoning' is related to reflective thinking toward problem-solving at the highest level. However, the relationship between proportional reasoning and reflective thinking toward problem-solving is negative. The relationship between reflective thinking and problem-solving is negative and significant with all variables.

Structural Model Predicting Proportional Reasoning Skill

MacKinnon (2008) explained that the mediation variable provides a causal explanation of the reflection between dependent and independent variables. The present study acknowledges that reflective thinking toward problem-solving is a mediating variable between self-regulation and proportional-reasoning skills. In the mediation analysis, three conditions were mentioned: the relation between the independent variable, mediator, and dependent variable; the relation between the mediator and dependent variable; and (when mediator was added to the model) whether the relationship between the independent and dependent variables was significantly reduced or not significant (Kline, 2011). This study set out to determine whether there was any relation between self-regulation and reflective thinking toward problem-solving and proportional reasoning. It then examined the relation between reflective thinking toward problem-solving and proportional reasoning.

The nonparametric bootstrapping method was used when the mediation hypothesis was tested. Studies recommend using the Sobel test to assess the significance of mediation in large samples. The bootstrapping method is used to resample smaller samples, in order to produce very large datasets (Field, 2013). In the present study, we reconstructed the sample size of 279 persons by 5000 with a 95% confidence interval using the bootstrapping method. In this way, we aimed to achieve more reliable results by analysing larger datasets generated by the resampling.

The analyses showed that the total effect of self-regulation on proportional reasoning was significant (β=-.53, t=-7.70, p=.00). We also determined that the effect of reflective thinking (β=-.65, t=-11.28, p=.00) and reflective thinking on proportional reasoning (β=.22, t=-3.07, p=.00) were significant. The absence of zero in the 95% confidence interval revealed that reflective thinking toward problem-solving played a mediating role between self-regulation and proportional reasoning. This mediated role had a high effect size (K^2=.30, β=-.14, 95% BCa CI [-.2510, -.0464]). Nevertheless, when the effect of reflective thinking was controlled, the predictive effect of self-regulation on proportional reasoning was still significant (β=-39, t=-4.72, p=.00), indicating partial mediation. Partial mediation shows that part of the relationship between self-regulation and proportional reasoning takes place directly, while the rest occurs indirectly through reflective thinking toward problem solving.

Figure 2 shows how the direct impact of self-regulation on proportional reasoning was added to the analytical model. In the model, standardised regression quotients are given.

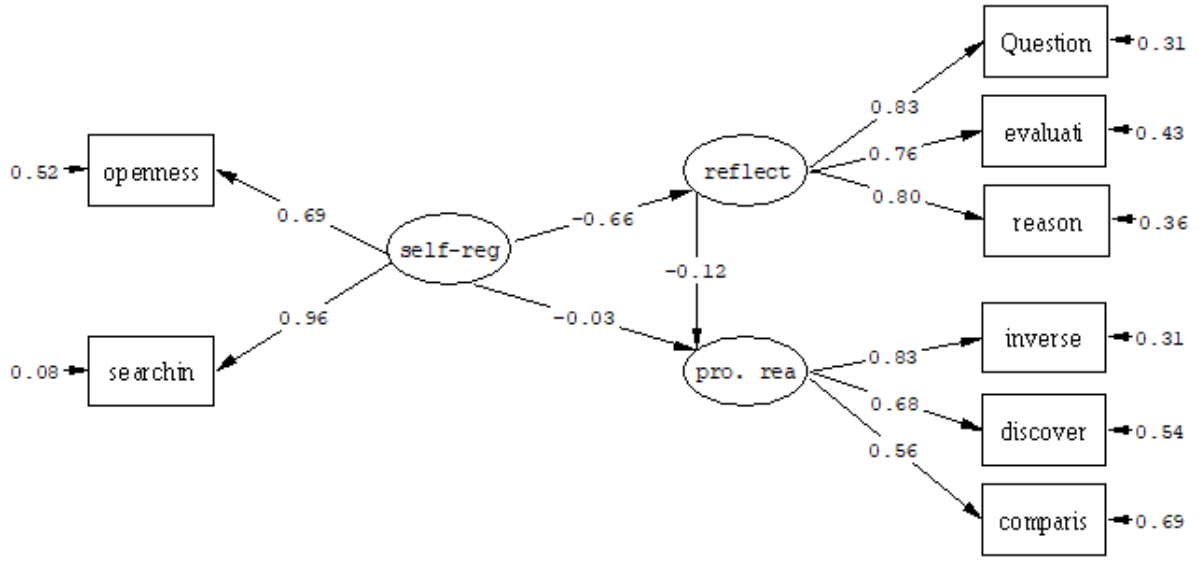

Figure 2. Structural Model Developed to Predict Proportional-Reasoning Skills

Discussion and Conclusions

This study assessed seventh-grade students to explain the relationship between reflective thinking toward problem-solving and self-regulation levels and proportional-reasoning skills. The study used structural equation modelling because its aims covered the cause-and-effect relation between variables and the mediation effect. The results of the correlation analysis revealed a significant relation between self-regulation levels, reflective-thinking skills toward problem solving, and proportional-reasoning skills. The mediation analysis revealed good compliance between the collected data and the predicted model. The overall results of the research show that proportional-reasoning skills can be predicted by taking the self-regulation levels of seventh-grade students into consideration. In addition, almost all relationships can be explained through reflective thinking. In other words, reflective thinking toward problem-solving partially mediates between self-regulation levels and proportional-reasoning skills.

The study results show the positive and significant relation between the self-regulation levels of seventh-grade students and their proportional-reasoning skills. Many studies have found that mathematical reasoning is related to self-regulation (Kramarski, 2008; Pape et al., 2003; Wolters & Pintrich, 1998). For this reason, the present findings on the relation between self-regulation and proportional reasoning support the literature. However, a significant negative relation has been found between self-regulation levels and reflective-thinking skills toward problem solving. Prior studies have explained the sub-dimensions, while explaining the theoretical relationship between these variables; a

positive relationship between these variables is expected (Tock & Moxley, 2017; Woolfolk-Hoy, 2015). Ghanizadeh's (2017) research on higher-education students found a significant relationship between reflective thinking and self-regulation. In the present study, the negative dimension between self-regulation and reflective thinking is in conflict with the literature. This divergence from the literature can be explained in the following way: first, the positive dimension is interpreted within the context of sub-factors; and second, the research used sample groups at different levels.

Similarly, the present study has found a significant relationship between reflective thinking toward problem-solving and proportional-reasoning skills. These results coincide with those of an early study conducted by Shin et al. (2003), which claimed that skills such as planning, monitoring, and regulating could be inversely related to problem solving. Riebman and Overton (1977) compared the proportional-reasoning skills of students who displayed reflective-thinking skills when solving problems with those of students who reflected less on the problems. They concluded that there was no significant difference between the students who did and did not reflect on the problems, when it came to proportional-reasoning skills. In our research, the relationship between the negative dimension of proportional-reasoning skills and reflective-thinking skills supports Riebman and Overton's (1977) results. According to Van de Walle et al. (2013), the fractions and rate concepts that underpin proportional reasoning help to develop reflective-thinking skills. Modestou and Gagatsis (2010) note that causation (justification) and evaluation skills are required for proportional reasoning. When reflective-thinking skills toward problem-solving are included in the analysis, self-regulation levels mediate proportional-reasoning skills at a significant level. Hayes and Embretson (2013) take the cognitive disorder between mathematical reasoning and self-regulation as a mediator variable. Their research deals with mathematical reasoning with a quantitative comparison dimension. The results indicate that cognitive disorder is a mediator between self-regulation, the surrounding disorder, test anxiety, and mathematical reasoning. In this mediating relation, the cognitive disorder relates to both mathematical reasoning and self-regulation with negative dimensions. Our finding – that the mediating effect of reflective-thinking skills toward problem-solving mediates the exploration of self-regulation levels by seventh-graders – supports the results reached by Hayes and Embretson (2013).

The present study has a number of limitations. First, the participants were selected using a purposeful sampling method from one city of Turkey. The second limitation involves the assessment of variables via scale. Combining qualitative and quantitative approaches would have shed light on quantitative relations and also produced an in-depth analysis of the process. For this reason, future researchers are advised to consider using mixed methods. The results of the study indicate that, among seventh-grade students, reflective thinking toward problem-solving mediates between self-regulation and proportional-reasoning skills. Future studies could contribute to the literature on proportional-reasoning skills by adding different variables and using different measuring instruments. Researchers in the future may manage experimental research toward examining proportional reasoning skills by trained about rate-proportion by learning with self-regulation. Practitioners should consider whether proportional-reasoning skills can be developed through rate-proportion training. At the same time, as the present study reveals, they should help students develop proportional-reasoning skills through practices that develop their self-regulation skills.

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