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Investigation of Students' Holistic and Analytical Thinking Styles in Learning Environments assisted with Dynamic Geometry Software [*]

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Abstract

The purpose of this study was to investigate the analytical and holistic thinking styles of 7th grade students who were taught polygons in a learning environment assisted with a dynamic geometry software (DGS) by using the Structure of the Observed Learning Outcome (SOLO) taxonomy. The study employed a qualitative action research design. The study group was composed of 16 seventh grade students. "The Problem-Solving Holistic and Analytical Thinking Scale" was administrated to determine whether students were analytical or holistic thinkers. In addition, the "Preliminary Level Assessment Examination" was administrated to reveal students' levels of comprehension of polygons. A three-week implementation was put into practice with the activities developed for the computer environment created by using the Geogebra software for the sub-learning domain of polygons. Following the applications, the "Final Level Assessment Examination" was administrated. In addition, the diaries were collected to determine students' opinions on this learning environment. Students' diaries were analyzed by performing content analysis. The data obtained from the Preliminary and Final Level Assessment Examinations were analyzed based on the the SOLO taxonomy. The results of the study revealed that the learning environment assisted with DGS had a positive effect on the learning processes of both analytical and holistic thinkers. Although the learning environment assisted with DGS demonstrated that students of either thinking style had upper-level responses in the solo taxonomy analysis, it has been determined that this dynamic environment did not account for any variation among students' thinking styles. It is recommended that such well-structured learning environments should be designed to provide students with different thinking styles and the ability to act in accordance with their own strategies.

Keywords

Secondary School Students
Analytical Thinking Styles
Holistic Thinking Styles
Dynamic Geometry Software
SOLO Taxonomy

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Introduction

Thinking ability is a unique feature that adds value to human beings, makes them more superior to other living beings and provides them with a feature that no other living being can possess. Developing this special competence of individuals, enabling them to use it actively, and raising individuals who are the subjects of their own learning processes (Ministry of National Education [MoNE], 2013) have gained priority in modern education. It is observed that individuals using active thinking, knowledge and skills seem to prefer their own ways of thinking or individual thinking prefences. This condition led to the emergence of the concept of thinking styles. The features that distinguish individuals from each other and can be described as characteristic styles of individuals express the preference, namely the style of the individuals.

The concept of style has emerged as a variable of individual differences in human performance (Sternberg, 1997; Zhang & Sternberg, 2000), and many new concepts have been produced under different structures related to this concept. Style is the way by which individuals choose to use something while doing or thinking about something (Sternberg, 1997). Individuals have the ability to use any form of mental representation they possess and they have the tendency to use one representation more intensively than others (Riding & Cheema, 1991). This orientation of an individual defines one's own style. Style refers to an individual's ability to solve problems, think, perceive, and remember in a way that is peculiar or familiar to him/her. In addition, style can be defined as individual differences that affect human performance (Riding & Cheema, 1991).

When the existing studies on style are examined, it can be observed that there are cognitive centered, activity centered (learning centered) and personality centered categorizations (Sternberg & Grigorenko, 1997). The "Theory of Mental Self Government" (Sternberg, 1997), which includes the three groups of styles described above, is a highly comprehensive concept in scope (Zhang & Sternberg, 2006). According to this theory, while individuals engage in a thinking style, they choose the most appropriate and convenient way of utilizing one's skill. In fact, with the influence of environmental conditions, most individuals are flexible in the thinking style they use, and the more flexible one is in changing the thinking style used, the more one is successful (Buluş, 2005). While Sternberg and Grigorenko (2001) defines individuals' styles as the way they prefer to do a particular task, he refers more to degrees rather than to certain categories of preferences. In other words, he mentions high and low levels based on the dimension of thinking that is addressed in the categorization of thinking style. Individuals can have many ways of thinking about a situation and seem to use their styles that are more dominant when compared to specific situations. Therefore, if an individual's task allows him/her to use their dominant styles, their chances of success in the task would also increase (Chaiyapornpattana & Wongwanich, 2012).

In Sternberg's theory (1997) *level* dimension, individuals are divided into two groups: global and local. Individuals whose local thinking style is dominant prefer to focus on details, specific and concrete examples. On the other hand, individuals whose global thinking style is dominant are found to enjoy the consideration of the whole by dealing with generalizations and abstractions. With Sternberg's distinction between local and holistic thinking styles, the concepts of holistic and analytical thinking styles used by different researchers to express their own concepts of thinking styles largely overlap.

Holistic and Analytical Thinking

Holistic thinking regards the problem as an incomplete object or picture and tries to complete it as a systematic and orderly structure, thus apply Gestalt psychology. Holistic individuals concentrate on details after looking at the general picture and understand the situation of the problem (Hammouri, 2003). Individuals who tend to have holistic thinking consider the effect which the whole has left in general. So they tend to process the entire information together. Individuals with a holistic thinking style do not focus on the parts of object, but approach the object as a whole in the first place. Therefore, they look at the big picture and get a general idea, paying little attention to detail. Individuals who have a holistic thinking style know the relationships between objects without focusing on details, and take into account the impact which these relationships have within the whole (Dewey, 2007).

Analytical thinking applies logical principles to achieve a result and uses indications that are more concrete. Students distinguish the problem from the different parts and features in attempt of reaching the result. It is expressed as an analytical strategy if students choose not to progress by segmenting the information process (Hammouri, 2003). Analytical thinking expresses how a single piece works in its entirety and what the effect of that piece is in its entirety (Dewey, 2007). Individuals with an analytical thinking style tend to process information linearly, regularly, and sequentially. In other words, they break up and analyze the information through individual analysis and decomposition.

In the literature, the relationship between thinking styles and other variables is examined comprehensibly. Grigorenko and Sternberg (1997) aim to determine the role of thinking styles in talent and academic achievement. A study on gifted high school students indicated no significant relationship between students' genders and thinking styles. In terms of talent, it was stated that thinking styles are related to academic achievement. It was also revealed that students with different thinking styles, but with the same level of thinking, showed better success in different forms of evaluation; no significant difference in terms of holistic and local thinking styles was reported. In addition, holistic and local styles was found to have no impact on academic achievement. Zhang (2003) studied the effects of thinking styles on the critical thinking ability of university students. According to the results of the study, there was no significant difference in students' thinking styles and critical thinking structures in terms of age, gender, class level and field of study. However, teaching that takes into consideration thinking styles influences the development of critical thinking structures because thinking styles influence critical thinking. Arıol (2009) conducted a descriptive study to determine the effects of holistic and analytical thinking styles of elementary school mathematics teacher candidates on mathematical problem solving skills. The study revealed that the thinking style did not show any meaningful difference in the problem solving ability of the teacher candidates. However, it pointed out that the individuals in the holistic thinking group produced unique solutions to some problems. The study emphasized that teachers should value students' thinking styles. Once it has been determined which of the analytical and holistic thinking styles are dominant, it is suggested that everyone should be trained in accordance with their own style. This type of education is expressed not as a way to classify students according to their thinking styles and to enable them to learn in different environments but as to know that they have both analytical and holistic thinking styles in the same class environment and to carry out their educational practices by considering this individual difference.

The idea that the learning environment and the applied teaching have an impact on thinking styles (Sternberg, 1994) and students’ ability to express their own thinking styles and reasoning during the problem solving process (MoNE, 2013) has motivated us to investigate how students with different thinking styles interact with teaching.

DGS and Holistic - Analytical Thinking

In the field of mathematics education, there is a variety of tools to enhance learning competency and to facilitate learning concepts and features. This area draws particular attention to dynamic geometry software (DGS), which is one of the tools commonly used to improve the quality of learning in mathematics, and geometry in particular. According to Güven and Karataş (2005), the DGS, which has shed a different light to the teaching of geometry, which has been continuing for years, provides support for experiences and teaching geometry to students through research. National Council of Teachers of Mathematics (NCTM, 2000) has been able to examine the characteristics of forms using DGS, to gain experience by making physical experiments and to understand relationships by using representations. According to Hazzan and Goldenberg (1997), it is possible to see DGS as a powerful tool that meets requirements in the learning environment. The forms created in this environment can move and change under various transformations that provide a flexible learning process. In this environment, students integrate their own learning principles with DGS (Baki, Güven, & Karataş, 2001). Knowing the thinking styles of students in this direction and designing a suitable teaching environment for these styles would be an effective process in terms of minimizing misunderstandings among students. This would strengthen classroom communication. The designed dynamic learning environment enables students to focus easily on abstract structures by offering alternative possibilities for paper pencil studies. This increases the potential for students to imagine. In mathematics, increasing the power of imagination means to open the path of intuition and therefore the path of discovery (Güven & Karataş, 2003). In other words, it would be appropriate to design a dynamic learning environment for individuals who are inclined to think in a holistic way of thinking, using their experience to take an advantage of them and using their answers in similar activities instead of following an intuitive path or stepping through the solution of the problem.

The use of computers in the process of making assumptions, testing, and generalizations among the basic elements of mathematics teaching allows students to have an idea about the mathematical results. In addition, like mathematicians, students are able to recognize what steps they take while achieving mathematical conclusions, contributing to the development of a unique style of thinking (Güven & Karataş, 2003). This suggests that it would be appropriate to design a dynamic learning environment for individuals with a tendency to think analytically, which also tends to divide and process the information in a sequential manner, and that apply logical principles to reach the problem. Thanks to the possibility that the dynamic structure presented, a learning process can be realized in accordance with the constructivist learning theory.

Individuals who use information in accordance with the style of thinking that they possess will participate in the learning process. It would show their talents in the process. It undoubtedly important that the process and the styles of thinking of individuals be considered in order to make the education and training process more efficient and effective. Teaching conducted in accordance with students' thinking styles would ensure permanent learning (Çubukçu, 2004). In this kind of an environment, there would be an opportunity to practice the cognitive structure of individuals. Numerous studies on computer assisted mathematics education (Moore, 2002; Tutak & Birgin, 2008; Hannafin, Truxaw, Vermillion, & Liu, 2008; Egelioğlu, 2008; Ubuz, Üstün, & Erbaş, 2009; Genç, 2010) have shown that students have been positively affected in terms of attitudes toward math (Chrysanthour, 2008; Genç, 2010; Musan, 2012). They increased their academic achievement levels by using computers in mathematics education in an appropriate way. In this context, the experience of students with different thinking styles in a learning environment assisted with DGS would reveal a positive outcome.

In the literature there are numerous studies employing different methods to evaluate the learning outcomes yielded by this kind of a learning environment. One of these methods is SOLO taxonomy, which has recently been utilized in many different studies, including the present study. SOLO has been used in the present study because it is found to be an effective evaluation tool in examining students’ responses to questions.

SOLO Taxonomy

The SOLO taxonomy (Structure of the Observed Learning Outcome) developed by John Biggs and Kevin Collis was developed as a general cognitive development model. This model was designed to assess students' cognitive skills and knowledge in relation to learning environments (Biggs & Collis, 1991; Lian & Idris, 2006). In the assessment process, the answers given by students to the questions could be analyzed according to the created scale (SOLO) and the SOLO taxonomy provided opportunity for the in-depth examination of these answers.

SOLO is a taxonomy developed by the reference of Piaget's cognitive developmental stages. Each stage was shaped by the logical framework that reveals its own characteristics (Biggs & Collis, 1991; Pegg & Tall, 2005). The developmental patterns of Piaget, Biggs, and Collis express that cognitive development has passed through certain stages and the developmental stages in both models are similar. The SOLO taxonomy consists of five cognitive stages (sensory-motor phase, imaginary phase, concrete symbolic phase, abstract phase, and post-abstract phase) and these phases are divided into

Piaget's cognitive developmental stages (sensory-motor phase, pre-abstract processes). The differences between these two models are the Biggs and Collis imaginary phase, which is the pre-process phase in Piaget's model. In addition, Biggs and Collis set out a new phase and expressed it as a post-abstract phase. Each thought phase forms the basis for the next phase. The previous thinking phase could be used by students to support their answers when they needed (Çelik, 2007). Both the SOLO taxonomy stages and Piaget's cognitive developmental stages appear based on age. Here, an important point that comes to our attention is that children could see at different stages in activities that can sometimes take place at the same stage. The SOLO taxonomy has been introduced here, and the SOLO taxonomy has been developed to overcome the inadequacy of Piaget's cognitive development model (Biggs & Collis, 1991; Pegg & Tall, 2005). Biggs and Collis focus on the answers they give, not on the levels of individual cognitive development (Pegg & Tall, 2005). This reflects the difference between the SOLO taxonomy and Piaget's cognitive developmental stages.

In the SOLO taxonomy, each thinking phase consists of five sub-domains called 'thinking levels' in each phase: (1) pre-structural (PS); (2) uni-structural (US); (3) multi-structural (MS); (4) relational (RS); and (5) extended abstract (AS). This classification is used to measure the quantitative and qualitative properties of the student's answers to any question. These two features together constitute structural complexity. The amount of detail in the perceptual cue given by the student reveals the quantitative direction of the structural complexity. The qualitative direction of structural complexity is how well the details that learners have listed in relation to each other. (Lucander, Bondermark, Brown, & Knutsson, 2010). According to this, while PS, US and MS levels of SOLO taxonomy reflect quantitative learning, the levels of RS and AS point to qualitative learning. The answers that reflect the qualitative direction of learners are signs of profound learning. Therefore, using the SOLO taxonomy, the students' answers could be analayzed quantitatively and qualitatively in terms of structural complexity (Leung, 2000).

According to Pegg and Tall (2005), it is expected that the individual has reached any of the levels of PS, US, MS and RS considering the training and individual differences that individuals have received during the learning process. A normal educational outcome cannot be achieved at the level of SY. This level can only be achieved with an effective teaching and learning process and with individual skills. This level is also a level associated with age 20 and above. Studies conducted in the field (Jones et al., 2000; Mooney, 2002; Bağdat, 2013) support this idea. For this reason, the SOLO taxonomy for the 7th grade students, which is the study group, was formed at four levels, PS, US, MS and RS.

Although the SOLO taxonomy is not a direct theory developed for mathematics, such as geometry thinking, algebraic reasoning, or realistic mathematics education, this taxonomy could be useful to evaluate the success of mathematics learning and teaching. The use of the SOLO taxonomy to evaluate mathematical success allows for objective evaluation. Moreover, it provides students with more meaningful feedback (Çetin & İlhan, 2016).

The Aim and Significance of the Research Study

Teaching that takes students' thinking styles into consideration would provide both efficient and permanent learning opportunities. It ought to be remembered that the thinking styles of students, who are the most important elements of education and training, are an important variable (Özbaş & Uluçınar Sağır, 2014). Since it is predicted that the analytical thinking style can be taught in the education system (Sternberg, 1994; Dewey, 2007), the thought that dominates the math lessons in the school setting is the mathematical problem-solving approach that progresses systematically in the analytical style. Indeed, the reason for this is that the analytical thinking is more useful in all matters, and the teaching of this style in the education system has kept it in the foreground. In the studies conducted to improve problem solving skills, the process considered is the steps of Polya (1973) to understand the problem, plan the solution, implement the plan, check the validity and validity of the solution, generalize the solution and establish a similar original problem (as cited in MoNE, 2013). As the idea of analytical thinking is more efficient, it is perceived that students are directed to this thinking style in primary, secondary and university class environments (Arıol, 2009). There is the holistic thinking style, which is expressed as the ability to come to mind in small details that do not hit the eye immediately in complex problems. Individuals with a holistic thinking style do not progress systematically in the solution of a problem or result from the controls. They prefer to find out the right result by using the answers they obtained in similar activities as an external source. Students who have such a thinking style are not considered valuable by their teachers in the educational setting. This style of thinking is suitable in the mathematical problem-solving approach that advances step by step in the analytical style (Hammouri, 2003), but this approach is not for holistic thinkers who try to solve the problem by looking at the overall idea of the problem. However, each individual in the holistic thinking style solves the problem in an intuitive way and with the help of similar examples of individuals with an analytical thinking style who break down the problem logically and solves it by using sequential processes, in the educational environment. Hence, it should be noted here that individual differences are not taken into consideration in the field of education when it is offered at a single level. Both analytical and holistic thinking are necessary and useful at the same time in the problem-solving process. If we consider the analytical and holistic thinking categories in a comprehensive way, they offer the comprehensive view of cognitive style, not just problem solving. It also includes the distinction between analytical and holistic thinking, information processing, and information gathering (Hammouri, 2003). Grigorenko and Sternberg (1997) suggest that an individual's thinking style is likely to increase the likelihood of success if the required thinking in the teaching and learning environment coincides with the style.

Rather than designing education based on a single thinking style, it is necessary to consider the fact that there may be individuals with different styles in these settings. Since the problem situations that individuals face in daily life are different from each other, it is necessary for individuals to have knowledge about different ways of solution in order to overcome the different problem situations. For instance, they are able to become aware of different styles and know how to use this style of thinking. According to İspir, Ay, and Saygı (2011), the identification of students' thinking styles is an important factor to shape learning. If we correctly define these thinking styles that students have and create a learning environments in which different methods and techniques are used, then the desired quality of education may be realized or achieved (Çubukçu, 2004). Duru (2004) states that the identification of students' thinking styles and the formation of appropriate teaching programs and teaching environments considering the styles would contribute to all the elements in education.

When differentiated teaching techniques are used, individuals learning in different ways may better be able to adapt to the educational environment. Education today is not only about the behavior of individuals, but also about change in the emotion and thought, supporting the learning environments that would reveal individuals’ potentials, and also provides individuals the freedom to choose the way they learn. The individual differences that arise in this situation concern an important area ranging from the regulation of the learning environments to the teaching methods and techniques used by teachers. Hence, individuals prefer different ways of learning, they have diverse minds, and each brain has its own unique structure (Esmer & Altun, 2013).

The purpose of the learning-teaching and assessment process is to include all students in this process and to monitor individual differences, which is one of the basic principles of the curricula (MoNE, 2013). Understanding the thinking styles would assist teachers in differentiating their teaching, that is, to consider individual differences, and as a result, to bring learning outcomes to the highest level (Sternberg, 1997; Sternberg & Grigerenko, 1997). In addition, this would also assist in measuring academic achievements accurately. For this reason, the identification of students' thinking styles is an important research topic in terms of education today. The present study aimed to examine, by means of the SOLO taxonomy, students’ analytical and holistic thinking styles in a learning environment assisted with DGS in teaching the topic of polygons to 7th grade students. In addition, students' opinions about this learning environment have also been revealed.

Method

In the present research, a qualitative research approach was adopted to collect, analyze and interpret data. An appropriate design was carried out qualitatively to reveal the reasoning underlying students' thoughts and to understand the process underlying the reasoning while constructing the response. This process cannot be determined from students' correct or incorrect answers to the problems. It is thought that it would be appropriate for teachers to assume a researcher role in effectively and efficiently progressing through the educational process in the study. The researcher teaching method is a research approach that involves the collection and analysis of systematic data developed in order to solve problems related to the teaching process or to solve a problem occurring in a special moment of the teaching process (Yıldırım & Şimşek, 2013). The role of teachers would be influential in the use of new methods in the classroom and in the overcoming of the problem encountered in practice. This method, which can be thought of as a tool for innovation in education, encourages teachers to conduct research in classroom practices (Altrichter, Posch, & Somekh, 1993).

Research Design

In the present research, initially, the sequence of the mathematical objectives followed in the mathematics course for the sub-learning domain of polygons was determined. While determining the sequence of the mathematics objectives, the researchers benefitted from the mathematics curriculum and studies on the sub-learning domain of polygons. The primary reason why the sub-learning domain of polygons was selected is its being among the fundamental concepts in geometry, the existence of literature on the features of polygons, and students’ misconceptions regarding the categorization of polygons. In the second stage, the 'Geogebra' software was decided to be used as a teaching tool. Activities and worksheets that needed to be used together with Geogebra were designed at this stage. While the activities were being prepared, it was noted that the features of the Geogebra software were available at the highest level. In order for students to use the Geogebra’s program at the desired level and in order to prepare for the application, the menus of the Geogebra program and its buttons were introduced to a computer-equipped mathematics class by the researchers for 1 week (7 lessons). During the presentation week, the course was taught by using activities and worksheets designed for the 'Direct and Open' sub-learning area prepared by the researchers.

In the third phase of the research, "The Problem-Solving Holistic and Analytical Thinking Scale" scale was used to determine students’ thinking styles. According to the result of this scale, it was determined that the students were closer to the analytical and holistic thinking style. Then, a "Preliminary Level Assessment Examination" (PLAE) was conducted to show the students' understanding of polygons. Later, the work that was designed in the computer environment by using the Geogebra software for the polygons sub-learning domain and the worksheets that helped them find out the mathematical structure that students had in each activity were applied to support these activities. Three field experts were consulted on whether these activities were appropriate for the polygon sub-learning domain. Fourteen activities were applied within a total of 21 hours for 3 weeks. Students constituted the social environment in which group work was the most appropriate place to express thoughts while building knowledge itself by making use of the possibilities provided by technology (Baki, 2008). With this in mind, the lessons processed in groups of three, with Geogebra so that each group would be able to use the computer. The “Final Level Assessment Exam” (FLAE) was performed after the application. FLAE was conducted in a dynamic environment where each student had the opportunity to use a computer.

Interviews were held with students when their answers were found to be unclear and incohesive in the PLAE and FLAE. Interviews were held to clear out the inconsistencies in the student responses and to find out how they thought about answering these questions. At the beginning of the interviews, the written answers of the students interviewed in the PLAE and FLAE were carefully examined by two experts. Accordingly, it has been decided which questions should be answered during the interview process. With the help of the data obtained through interviews, the levels of the students could be determined easily. The obtained data were evaluated under the relevant question of the interviewed student. The data obtained from PLAE and FLAE were evaluated according to the SOLO taxonomy.

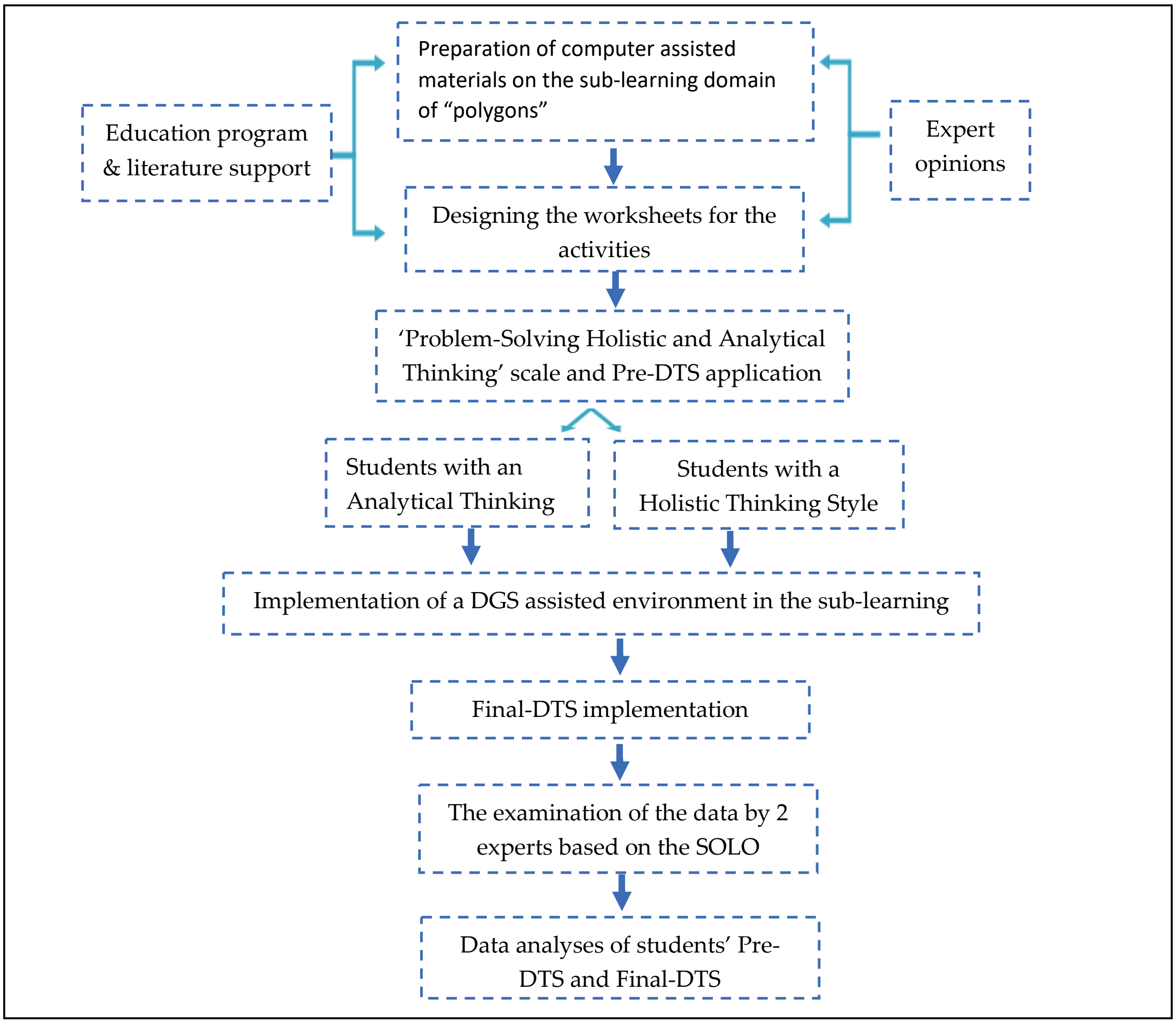


Figure 1. Flow Chart of the Research Study

Research Group

The research group consisted of 24 middle school students who were successful in the middle level in Rize province center and 7th grade students. The "Problem-Solving Holistic and Analytical Thinking Scale" was used to determine which thinking style – analytical or holistic thinking style – was dominant among 7th grade students. On this scale, a minimum of 1 and a maximum of 3 points could be received from each item. Therefore, for the five items, a minimum score of five and a maximum score of 15 could be received. Those with an analytical thinking style receive a maximum of 5 points, while those have a holistic thinking style receive a maximum of 15 points from the entire scale. As a student gets closer to the score of 5, s/he seems to be closer to the analytical thinking style. If the student's score is closer to 15, s/he is believed to be closer to the holistic thinking style. The scores of the students in "The Problem-Solving Holistic and Analytical Thinking Scale" is presented in Table 1.

Table 1. Scores from "The Problem-Solving Holistic and Analytical Thinking Scale"

Students with an analytical thinking style								**Students with a holistic thinking style**			
Scores	5 score	6 score	7 score	8 score	9 score	10 score	11 score	12 score	13 score	14 score	15 score
Number of Students	-	1	5	2	3	1	4	1	7	-	-

According to Table 1, the students within the score range of 5-8 are observed to be closer to the analytical thinking style and those within the score range of 12-15 are found to be closer to the total thinking style. It has been found that 8 of the 24 students were analytical thinkers and another 8 were holistic thinkers. The remaining 8, on the other hand, could not be placed within any one of the thinking styles. Analytical thinking students were coded as Sa_1, Sa_2, Sa_3, Sa_4, Sa_5, Sa_6, Sa_7, Sa_8, while the holistic thinking students were coded as Sb_1, Sb_2, Sb_3, Sb_4, Sb_5, Sb_6, Sb_7 and Sb_8.

Designing the Activities

Activities were designed in the sub-learning domain of dynamically structured "Polygons" prepared with the assistance of the Geogebra software to be used in class. During the development of these activities, mathematics curricula, mathematics textbooks and research studies in this area were benefitted from. Subsequently, supplementary worksheets were prepared in order to help students reveal the mathematical structure in each activity. Two expert opinions were received to understand whether or not the prepared activities were appropriate for the sub-learning domain of polygons. One of the experts was a lecturer, and the other was a teacher with 6 years of teaching experience. The method followed in preparing the activities is as follows:

Special attention was paid to

- enabling individuals to construct knowledge themselves rather than transferring the knowledge to them,
- integrating the knowledge into the activities in a planned way to arouse curiosity,
- preparing open-ended questions for group work for the discovery of the target feature to be learnt,
- ensuring that the solution is found by the students with the help of clear and comprehensible instructions and that they do not feel the need to frequently ask questions to the teacher,
- providing the students with the opportunity to discuss and question the solutions initially in groups and then in a whole class discussion (Baki, 2002).

The activities were designed in such a way that they gave students the opportunity to gain their own experiences and construct their own knowledge. The aim was to enable students to use their own thinking styles during the activities.

Data Collection Tools

Student diaries, the Holistic and Analytical Thinking Scale, the Preliminary Level Assessment Examination (PLAE) and Final Level Assessment Examination (FLAE) constitute the data collection tools of the study aimed at examining the effect of the learning environment in which DGS was used to teach polygons to 7th grade students who had a holistic or analytical thinking style. The diaries informed the researchers about how individuals experienced the teaching environment and about the learning process. Students were asked to write in their diaries at the end of the day after their lessons performed in a learning environment created by using DGS.

A portion of the data was generated in the study obtained from PLAE and FLAE. The DTS questions were developed by using the related literature, TIMMS and PISA questions, and textbooks approved by the Ministry of National Education. Problems were designed by means of the method of document review. Three field experts were consulted on whether these problems were suitable for the polygon sub-learning domain and the final form was given after the pilot implementation. Based on the expert opinions, the PLAE consisted of grade six objectives taking into account the students' readiness levels, while The FLAE consisted of questions addressing grade 7 objectives. SOLO taxonomy levels were utilized to obtain more concrete results than the data obtained from these problems.

Holistic and Analytical Thinking Scale

"The Problem-Solving Holistic and Analytical Thinking Scale" developed by Arıol (2009) was included in this research to reveal the students' thinking styles. The items that make up the scale are based on the features of holistic and analytical thinking styles in the literature and are believed to represent the effects of these styles on problem solving skills. There are five items on the scale and these items intend to show whether students are closer to analytical or holistic thinking. Each item has a problem-solving expression that matches both the analytical and holistic thinking style in which individuals can express their preferences in problem-solving situations. One of the items in the "The Problem-Solving Holistic and Analytical Thinking Scale" is as follows: "While solving a problem, I immediately envisage the solution" and "While solving the problem, the solution generally develops during the process." Students were asked to choose the appropriate one for these two different situations, and the "No idea" option was chosen by those who were unable to identify their thinking style (Arıol, 2009). During the development of the scale, first expert opinion was sought, and then item analysis was performed. Subsequently, the reliability coefficient was calculated and the scale was finalized (Arıol, 2009). The reliability coefficient of the scale was calculated to be 0.78.

Student Diaries

The diaries kept during the research study provided the researchers with beneficial contributions in understanding the participants' observations, feelings, interpretations and explanations (Yıldırım & Şimşek, 2013). Diaries enabled the researcher to gain insight into how the participants experienced the learning environment and the learning processes. They had a positive impact on participants' interaction with the learning environment, on their self-observation during the implementation, and on the increase in their self-awareness. In the diaries utilized in the present study, the students were asked to answer such questions as the following: "What have we done in today's lesson? Explain briefly", "What parts of today's lesson raised curiosity in you, excited you, or influenced your learning?" The students were asked to write in their diaries at the end of the lessons conducted in a learning environment assisted with DGS. Each student wrote a total of 10 diary entries. The data obtained from the each student's diaries were read thoroughly and based on their responses in their diaries, three themes were constructed. These were cognitive, affective and software program. Thus, students' opinions regarding lessons in a learning environment assisted with DGS could be evaluated. Accordingly, initially, the opinions of the whole class regarding the teaching of polygons in a learning environment assisted with a dynamic geometry software were received. Subsequently, for comparison purposes, those students who possessed an analytical or holistic thinking style were consulted for their opinions regarding the teaching of polygons in a learning environment in such a learning environment.

Preliminary Level Assessment Examination (PLAE) and Final Level Assessment Examination (FLAE)

Some of the data obtained in this study were derived from the DTS. During the development of the DTS questions, studies in the literature, items in the TIMMS and PISA, and textbooks approved by the Turkish Ministry of National Education (MoNE) were benefitted from. By means of document analysis, subject related sections were reviewed in order to construct the problems. Field experts were consulted to determine whether or not these problems were suitable for the polygon sub-learning domain and the final form was given after the pilot implementation. The aim of the first question in PLAE is to examine whether or not students can solve problems on the relation between side length and area of polygonal figures. In the sub-items of this problem, the students are asked to state the change in area as the difference between the side lengths of a rectangular figure becomes smaller. They are also expected to design new rectangular figures with areas twice as much as the area of the one given to them and finally, they are expected to draw a new triangle with an area twice as large as that of the rectangle. In another question, students' ability to solve problems on the relation between perimeter lengths and areas of polygons is examined. In the sub-items of this question, the students are asked to state the change that occurred in the perimeter lengths as the areas of polygonal figures becomes larger; they are expected to state that the length of a perimeter changed based on the shape of the figure and construct a general statement about a geometric figure that can be constructed with the largest perimeter length. In the last question, students' problem solving skills in questions related to areas of polygons were examined. In the sub-items of this question, the students are expected to state how many triangles are needed to cover the given rectangle and draw the new perpendicular triangle that is asked for. As in the PLAE, the FLAE also includes the same distribution of questions. The first question of the FLAE is related to a problem on the relation between the side length and area of polygons. In the sub-items of this problem, the students are expected to state the change in the area of a rectangle when the difference between the short and long length becomes smaller, to decide what kind of a relation there would be among the given length sides of the rectangle in order to have a rectangle with the largest area, and subsequently state that the side lengths need to have values very close to each other. In another question, a problem has been prepared addressing the relation between the perimeter length and area of polygons. In the sub-items of this problem, students are required to use more than one piece of data, explain the relation between the data and the entire question, and recognize how much the perimeter changes as the size of the area increases. Finally, a problem related to the area of polygons has been included. In the sub-items of the problem, students are required to find the areas of different tetragonal figures and pay attention to the relation among the areas of these figures. A sample question in the DTS is presented in Figure 2.

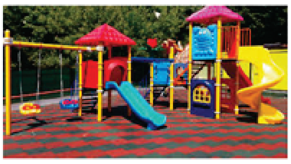
Data Analysis

In this research, rubrics were needed to objectively analyze the data obtained from PLAE and FLAE prepared for polygons. Rubrics were prepared taking into account the SOLO taxonomy. An expert was familiarized with SOLO taxonomy to accompany the researcher in the study. The data obtained in the pilot study were assigned to the respective level of the students, taking into account the definitions of the SOLO taxonomy levels, independently of each other. After pilot work, minor corrections were made to the rubric.

Students were interviewed by the researcher to clarify student responses that were not understood in the PLAE and FLAE. With the help of the data obtained through the interviews, student levels could be determined easily. Thus, the data were evaluated under the relevant question of the interviewed student. The internal validity of this research was ensured. The leveling process was completed again according to the new situation. After the leveling process was completed, the double-coding method defined by Miles and Huberman (1994) was used to establish inter-rater reliability. Accordingly, a compliance of 88% was achieved. As Miles and Huberman (1994) pointed out that 70% or more of them showed reliable coding, it was conceivable that the scale developed for leveling SOLO Taxonomy would be suitable for a consistent and reliable leveling.

Below is a description of how the leveling of possible answers to the students in the PLAE and FLAE would be (see Figure 1).

1.Okul müdürümüz, okul bahçesine yapılacak oyun parkı için bir tasarım yapmanızı istiyor. Oyun parkının çevresi 30 metre olacak şekilde dikdörtgensel bir tasarım oluşturmalısınız.



a.Uzun kenar uzunluğu 13 m,kısa kenar uzunluğu 2 m olan dikdörtgensel oyun parkının alanı kaç m^2 dir?

b.Dikdörtgensel oyun parkının kenar uzunlukları birbirine yaklaştıkça alanın nasıl değiştiğini açıklayınız.

Uzun Kenar	Kısa Kenar	Alan
14 m	1 m	$14 m^2$

c.Tasarladığınız herhangi bir dikdörtgensel oyun parkının kısa kenar uzunluğu 2 katına çıkarılıp uzun kenarının uzunluğu yarıya indirilirse dikdörtgensel bölgenin alanında nasıl bir değişim olur?

d. Okul müdürümüz, okulun bahçesine yapılacak kum havuzu için bir tasarım yapmanızı istiyor. Kum havuzunun kenarları keresteler kullanılarak yapılacaktır. Sizden uzunlukları 3, 8, 10 ve 11 m olan dört kereste kullanılarak dörtgen şeklinde, alanı en büyük olacak şekilde kum havuzunu tasarlamanız isteniyor. Bunun için keresteleri hangi uzunluklarda parçalara nasıl ayıracağınızı açıklayınız.

Figure 2. Sample Question from PLAE and FLAE

The possible answers that can a student can give to the question in Figure 2 are portrayed in Table 2. The SOLO taxonomy levels corresponding to these responses and the possible conclusions regarding the level of understanding of students for each response can be observed in Table 2.

Table 2. Application of the SOLO Taxonomy to a Question in PLAE and FLAE

Student Response	Level	Implications of Student Information
He could not find the area of the rectangular figure.	PS	The student did not understand the question.
He tried to find the area of the rectangular figure by doing random operations.	PS	The student did not understand the question.
He found the area of the rectangular figure in problem item a, and in question b used the expression "The area of the rectangle approximates the length of the short side to the long side".	US	The student noticed that as the length of the rectangle increases the side lengths of the rectangle converge, the products of the side lengths increase the area by a certain number. In other words, the student at this level has reached the essence by focusing on one aspect of the problem situation.
Responding to question a and b; In item c, for example; If the side length = 8 cm, the side length = 10 cm, then Area = 80 cm^2 and then the side length = 8 cm x 2 = 16 cm, side length = 10 cm: 2 = 5 cm Area = 80 cm^2.	MS	The student has used multiple bibliographies to solve the problem, increasing the short side length of the rectangle 2 times, reducing the long side length to half; "The area does not change."
Answered the question a, b, c. In item d, 2 m from 10 m timber, 3 m from 11 m timber are cut and these pieces are attached to 3 m timber given to the question (3 m + 3 m + 2 m = 8 m). Since the other timbers are 8 m, the maximum area is 64 cm^2.	RS	The student conceives all aspects of the problem in the case of problems, their place in the problem, and other related connections. In question d, he decided how to relate the side lengths in order to make the rectangles with the greatest area with the given lengths, then he could design rectangles thinking that they needed to have the closest value to each other.

The SOLO taxonomy levels of the potential student responses to a question on the relation between the side length and area of polygons are presented above. The diaries, another data collection tool, were filled out by the students at the end of the last lesson conducted in the DGS learning environment. In the analysis of student diaries, content analysis, one of the qualitative analysis methods, was used. Content analysis is a systematic technique where the data are summarized with smaller content categories with the coding based on certain rules (Büyüköztürk, Çakmak, Akgün, Karadeniz, & Demirel, 2013). Each student wrote a total of 10 diary entries. First the diaries collected were given a number and then the responses of the students were categorized under three themes: cognitive, affective and software-related. The data under these themes were coded by two field experts independently. The disagreements between the coders were eliminated by negotiation and based on common opinion, they were placed under the relevant theme. These coded data were presented in a table format and the student numbers and percentages, as well as sample student opinions are presented. In addition, in the results section, there are direct quotations from student diaries. The holistic and analytical thinking scale was used only to categorize students according to their thinking styles, and how the data obtained from the scale were analysed is explained in detail under the title "research group".

Results

Initially, the findings of the evaluation via the SOLO taxonomy of the responses given to the PLAE and FLAE by the students who were categorized into two groups – those with a holistic and an analytical thinking style – are reported.

SOLO Taxonomy Assessment of the Responses Given to Each Item in PLAE and FLEA by the Students with an Analytical Thinking Style

Analyses were made using the SOLO Taxonomy of the responses given to PLAE by 7th grade students with an analytical thinking style prior to the implementation of a dynamic learning environment and of responses to 3 different problems in FLAE after the implementation. The result of the evaluation based on the SOLO Taxonomy of the responses given to each question is presented in Table 3 below.

Table 3. Evaluation of the Responses to the PLAE and FLAE Items Based on the SOLO Taxonomy

Student	Question 1: Side Length-Area Relation		Question 2: Perimeter Length- Area Relation		Question 3: Areas of Polygons	
	PLAE	FLAE	PLAE	FLAE	PLAE	FLAE
Sa_1	RS	MS	US	MS	US	US
Sa_2	MS	MS	US	MS	US	US
Sa_3	US	PS	PS	US	PS	US
Sa_4	US	MS	US	US	MS	US
Sa_5	PS	US	US	US	PS	PS
Sa_6	US	MS	US	MS	US	US
Sa_7	PS	US	PS	US	US	PS
Sa_8	PS	PS	US	PS	US	PS

Upon the examination of Table 3, it can be observed that in the PLAE there are 3 PS, 3 US, 1 MS student in the question on side and area relation of polygon figures, 2 PS and 6 US students in the question regarding perimeter length and area relation of polygonal figures, and 2 PS, 5 US and 1 MS student in the question on the area of polygons. On the other hand, in the FLAE, there are 2 PS, 2 US and 4 MS students in the question on side length-area relation of polygonal figures. There are 1 PS, 4 US and 3 MS level students in the question about perimeter length and area relation of polygonal figures and 3 PS and 5 US students in the question about areas of polygonal figures.

The data derived from the PLAE and FLAE responses of the 7th grade students with an analytical thinking style regarding the relationship between side length and area of polygon figures were compared. According to the SOLO taxonomy levels, in the PLAE there were student responses at the pre-structure and uni-structural phases, while in the FLAE, it was observed that there was a transition towards multi-structural level, which is a higher level phase. In this question, many of the students could use more than one data to explain the changes in the side lengths of the rectangle; however, it was observed that they remained to be incompetent in deciding what kind of a relation there should be among the side lengths to obtain the largest area. In another question on the relation between perimeter length and area of polygon figures, it was observed that the responses of 7th grade students with an analytical thinking style were primarily at the stage of a single-directional structure in the PLAE, while a transition towards a multi-directional structure of the SOLO taxonomy thinking levels was observed in their responses to the questions in FLEA. That is, it was observed that these students used more than one data while arriving at their responses. The students realize to what extent the perimeter length changes when the area increases and arrive at the solution in a practical way. The data derived from another question, on the area of polygon figures, revealed that 7th grade students with an analytical thinking style were primarily at the level of uni-directional structure of the SOLO taxonomy in both

PLAE and FLAE. Thus, it was observed that students with an analytical thinking style maintained their thinking levels regarding areas of polygon figures and were able to find the areas of tetragonal figures with the help of concrete data; however, they experienced difficulty in using more than one data in the field of quadrilateral areas.

Sample student responses in relation to SOLO levels are provided below. Figure 3 presents the response of the Sa$_1$ coded student to a problem in FLAE on the relation between side-length and area in polygon figures.

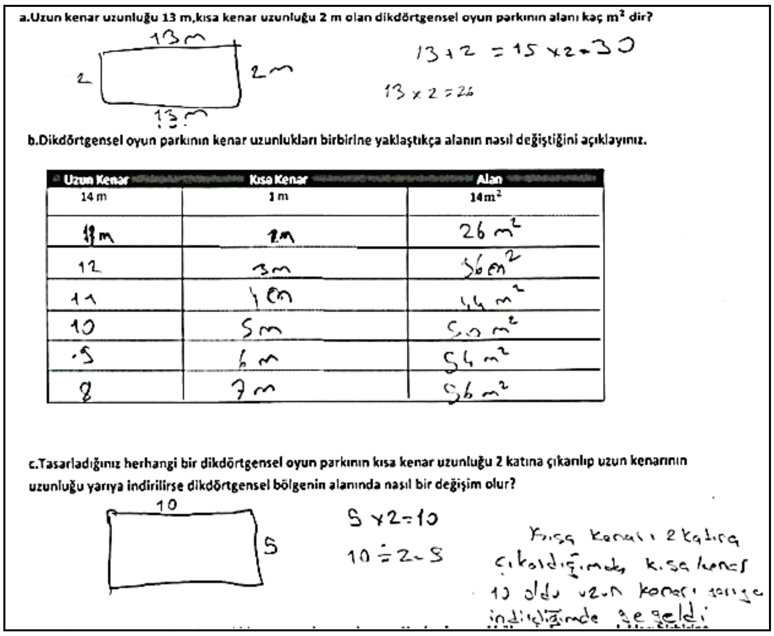


Figure 3. The Response of the Sa$_1$ coded Student

In the figure, the Sa$_1$ coded student found the area in item a of the problem accurately and mentioned the change in area in item b. However, the response to the "How?" component of the instruction in item b – "Explain how the area changes as the side lengths of the rectangular playground come closer to each other." – could no be seen on the DTS paper. Hence, a face to face interview was held with the student.

Teacher: Why have you written 13x2 = 26 to item a of the first problem?
Sa$_1$: It asked for the area of the rectangle, so I multiplied the long and short sides, that's why I wrote that.
Teacher: Okay, you haven't answered the question in item b: "How does the area change?"
Sa$_1$: I forgot to write the answer…
Teacher: What should the answer be?
Sa$_1$: *It increases, I mean the area increases.* I first found 14, then it *increased* to 56.

In the interview, the Sa$_1$ coded student stated that area would increase in item b of the problem. The student was able to display the ability to use more than one piece of data as expected in item c. However, the written verbal statement on the student's level assessment exam paper was incomprehensible. This incomprehensible part was clarified during the interview.

Teacher: What did you write for item c of the problem, I couldn't read it, can you read it?
Sa$_1$: It remained *the same*… I mean it *doesn't change*, that's what I wrote.

The student reached the expected answer in item c of the problem as well. Hence, the Sa$_1$ coded student was determined to be at the MS level. As the Sa$_2$, Sa$_4$ and Sa$_6$ coded students gave similar answers, they were assigned to the MS level.

The Sa_6 coded student's response to a question on the area of polygonal figures in FLAE can be observed in Figure 4 below.

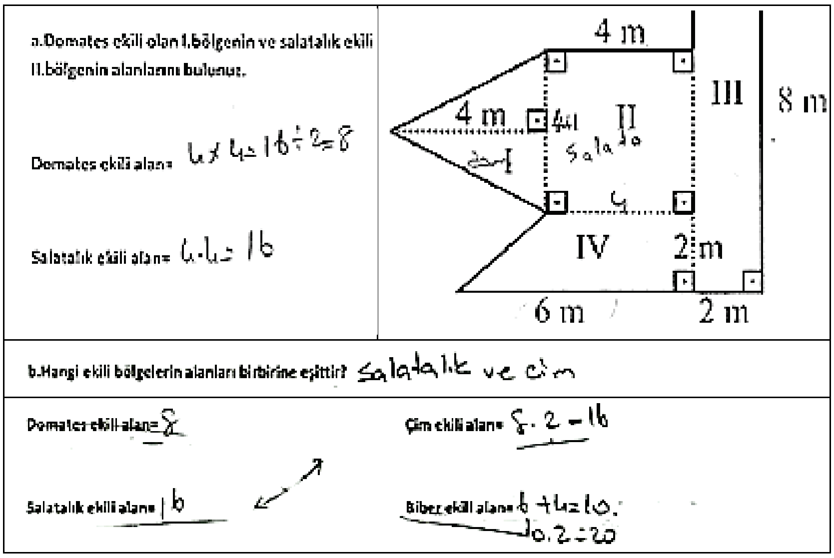


Figure 4. The Response of the Sa_6 coded Student

In Figure 4, student Sa_6 has calculated the areas of the tetragonal figures in item a, and calculated the areas of each tetragonal figure given on the figure to find the areas equal to each other in item b. On other hand, the student gave inaccurate responses to items c and d and did not use more than one piece of data in responding to the question. Thus, the student was assigned to level TY. Sa_1, Sa_2, Sa_3 and Sa_4 coded studuents gave similar responses, so they were also assigned to level TY. The responses of the other students were also evaluated in the same way and assigned to the corresponding levels.

SOLO Taxonomy Assessment of the Responses Given to Each Item in PLAE and FLEA by the Students with a Holistic Thinking Style

A SOLO taxonomy analysis was performed for 7th grade students with a holistic thinking style, PLAE before application in dynamic learning environment and FLAE after application. The findings of the SOLO taxonomy assessment of the responses given to each question is presented in Table 4.

Table 4. Solo Taxonomy Assessment of the Responses given to PLAE and FLAE Questions by Students with a Holistic Thinking Style

Student	Question 1: Side Length-Area Relation		Question 2: Perimeter Length-Area Relation		Question 3: Areas of Polygons	
	PLAE	FLAE	PLAE	FLAE	PLAE	FLAE
Sb_1	US	MS	US	US	MS	MS
Sb_2	US	PS	PS	US	PS	US
Sb_3	US	MS	US	US	PS	MS
Sb_4	PS	US	US	US	US	PS
Sb_5	RS	MS	US	RS	US	MS
Sb_6	PS	US	PS	PS	PS	US
Sb_7	PS	US	US	PS	US	US
Sb_8	RS	MS	MS	RS	MS	MS

As presented in Table 4, there are 3 PS, 3 US, and 2 RS students in the PLAE for the side length and area relationship of polygonal figures, while there are 2 PS, 5 US and 1 MS student in the question on the relation between perimeter length and area of polygon figures, and there are 3 PS, 3 US and 2 MS level students in the field of polygonal areas. There are students at the level of 1 PS, 3 US and 4 MS in the question about side length and field relation of FLAE polygonal regions while there are 2 PS, 4 US, 2 RS students in the question about environmental length and field relation of polygonal regions. There are the 1 PS, 3 US, 4 MS level students.

The data regarding the relation between side length and area in polygonal figures obtained from the responses given to PLAE and those given to FLEA by 7th grade students with a holistic thinking style were compared. It was found that based on the SOLO taxonomy levels, there were student responses at the pre-structure level and at the uni-structural level in the PLAE. The most striking feature of the students at the pre-structure level was that their responses were either insufficient or irrelevant. As for the students at the uni-directional level, considering that they have a uni-structural thinking level, it is understood that they could not respond by considering all the apsects of the questions. Hence, it was found that they had limited knowledge of the subject. However, it was found that they had given a high ratio of responses at the level of the multi-structural of the SOLO taxonomy in the FLAE. The assessment of the responses given to another question, on the relation between perimeter length and area in polygonal figures, by 7th grade students with a holistic thinking style, revealed that the responses in the PLAE were primarily at the uni-structural level, while those in the FLAE were also at the uni-structural level but there were also indications of transitions to a higher level – the relational structure level. It was observed that students at the level of uni-directional structure did not experience difficulty in focusing on the problem but only used a single piece of data to solve the problem. They were unable to recognize the place and relation the single piece of data they used in the problem had within the entire data. The FLAE results revealed that there were students at the relational structure level as they could combine data to make generalizations, and understand the relationships among data. In another question, on the area of polygonal figures, the responses of the 7th grade students with a holistic thinking style in the PLEA revealed that they were dispersed across the levels of pre-structure, uni-directional structure, and multi-directional structure. The most striking feature of the students at the pre-structure level was their insufficiency of their responses. It is striking that their responses are far from the features of the stage they are at. Considering that students at the level of uni-directional structure have a uni-structural thinking system, it can be understood that they cannot provide responses by taking into consideration all the aspects of a problem case. Hence, it was concluded that they had limited knowledge of the subject. In the FLAE, it was observed that there were transitions to multi-directional structure of the SOLO taxonomy thinking stages as there were more responses at the level of multi-directional structure. It was observed that students used more than one piece of data in responding to questions about the areas of quadrilaterals, thus indicating a multi-directional structure level. Considering a problem from a wider perspective and evaluating and stating the potential conditions require a relational structure level; there were not responses displaying this level.

Sample student responses displaying various SOLO levels are provided below. Figure 5 presents the response of a student coded Sb_{24} to a PLEA problem based on the relation between the perimeter length and area of a polygonal figure.

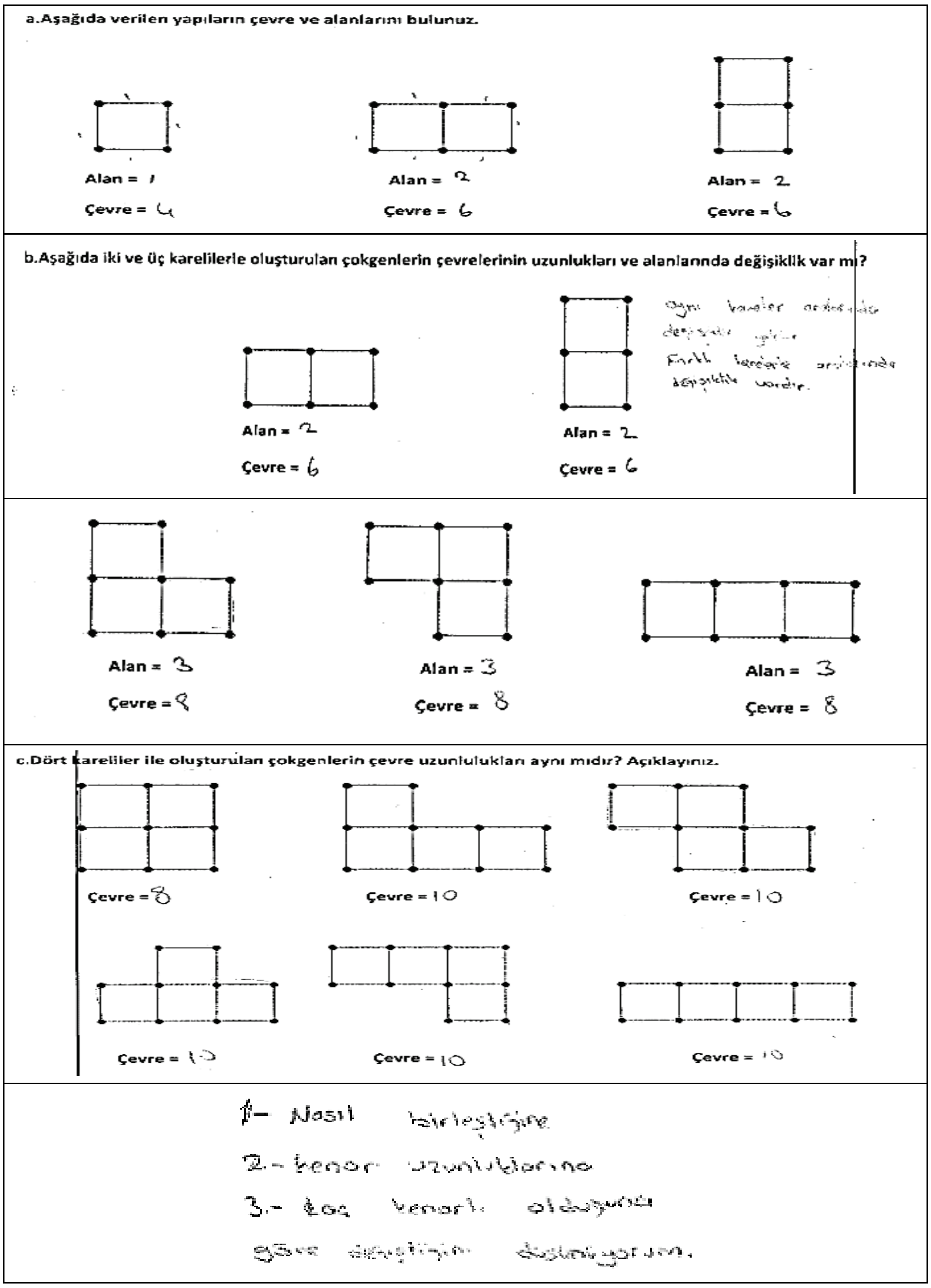


Figure 5. The Response of the Sa_8 coded Student

Sa_8 coded student correctly answered item a, in item b of the question "There is no change between the same frames, there is a change between different frames." expressed the change. In item c and item d of the problem, the answers of the student are given below. In item c of the problem, the students were asked to comment on the circumference of the formed quadrilaterals. Student Sa_8 coded " I think that the shape changes depending on how it joins, edge lengths, how many edges." statement on the PLEA. the same student gave the wrong answer in item d. Sa8 coded student used more than one data related to the answer; however, these data were found to be insufficient to explain its relationship with the whole problem. For this reason, Sa8 coded student has been assigned to MS level.

In FLAE, in the question regarding the relationship between the perimeter of the polygonal regions and the area, the answer of Sb_5, one of the students with holistic thinking style, is given below.

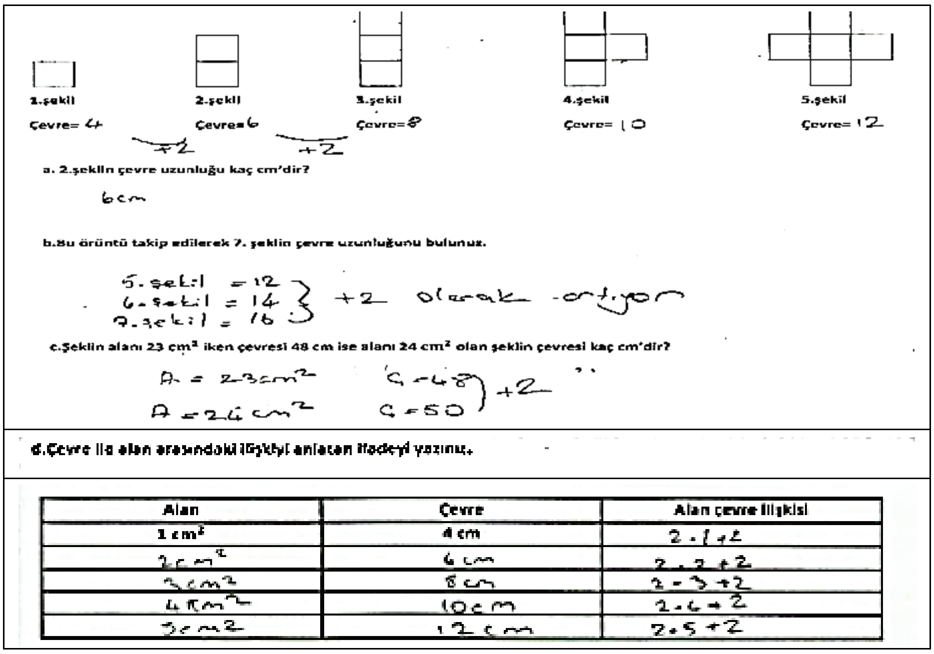


Figure 6. Sb_5 Coded Student's Response

In Figure 6, the Sb_5 coded student has arrived at the answer by taking into consideration the amount of increase in the pattern of the first two items and, by applying the same strategy to item c, found the perimeter of the figure to be 50 cm. It is accepted that RS level is achieved by integrating the information given in item d to reach a generalization and indicating that the area is two plus two times the area. Giving similar response, Sb_{24} coded student was assigned to RS level as well. The findings of the differentiation between students with holistic and analytical thinking styles in a learning environment assisted with DGS are reported below.

Dynamic Learning Environments of Students with Holistic and Analytical Thinking Style Before and After Lesson SOLO Taxonomy Levels

The study attempted to reveal whether or not a learning environment assisted with DGS created any variation in the SOLO taxonomy levels of students with analytical or holistic thinking styles. Thus, the distribution of the holistic and analytical thinking students' responses to PLAE according to the SOLO taxonomy is presented in Table 5.

Table 5. Distribution of Students' Responses According to the Solo Taxonomy in PLAE

	PS	US	MS	RS
Analytical Thinking	7	14	2	1
Holistic Thinking	8	11	3	2

When the Table 5 is considered, seven of the answers of students with analytical thinking style are PS, fourteen are the US, two are MS and one is RS. Student responses with a holistic thinking style are eight PS, eleven US, three MS and two RS. It was observed that students of both thinking styles are predominantly in the US in the SOLO taxonomy phase. This indicates that 7th grade students with analytical and holistic thinking style are close to each other in terms of preliminary knowledge of the subject. The Table 6 shows how students with analytical and holistic thinking styles differed from the SOLO taxonomy with answers to the FLAE after implementation.

Table 6. Distribution of Student Responses by Solo Taxonomy in FLAE

	PS	US	MS	RS
Analytical Thinking	6	11	7	-
Holistic Thinking	4	10	8	2

Six of the responses of students with analytical thinking style according to the Table 6 are PS, eleven in the US and seven in the MS level, and no response at the RS level. Likewise, four of the answers of students with a holistic thinking style are the PS; it is the US, eight MS and two RS. In the case of the findings obtained, students with both types of thinking informed that they are at US and MS levels in SOLO taxonomy thinking stage. Therefore, it was observed that the learning environment created by using dynamic geometry software in the teaching of polygons does not make a difference in the 7th grade students who have analytical and holistic thinking style.

Student Opinions about the Learning Environment Created Using Dynamic Geometry Software

Students' opinions about teaching mathematics in a dynamic environment obtained from the diary that the group filled in after the events during the practice. Each student has written a total of 10 days. Three different themes were identified, with the student logs related to the cognitive, emotional and software programs and student opinions were analyzed according to these themes. It indicates that students have positive opinions regarding the learning environment.

Table 7. Student Opinions about the Learning Environment Created Using Dynamic Geometry Software

Theme	Codes		Analytical thinking f	Holistic thinking f
Cognitive Views	Better understanding in a computer-based environment/ better mindfulness /easier to learn	*... making use of the computer makes our work easier and makes sure that the formula is not memorized... makes things harder, keeps better in my mind, the lesson is fun/ ... I have understood the issues better.*	6	6
	Self-assisted learning in a computer-based environment /discovering with the help of the teacher/ self-discovery	*... we discovered the properties of polygons, we found them with the help of the teachers/ ... we found the formulas of the area of the polygons ourselves/…we learn by self-discovery, it was an interesting lesson for me, …I wish we could always learn by discovery.*	3	4
Affective Opinions	The computer is more fun in the environment used/ more interesting	*... it made it more fun, enjoyable and interesting. It is better this way because we learned by having fun/ … it was a fun class. I liked it very much. I would want it again.*	6	7
	Being fond of the visuality addition by the computer	*...we did the topic from the computer in a clearer and more visual way/... it adds visuality to the lesson so I learn a difficult topic easily. And I like this…*	2	4

Table 7. Continued

Theme	Codes		Analytical thinking f	Holistic thinking f
Software Opinions	Geogebra software helps to find errors more quickly	*... I understand my mistakes better/…using the computer in mathematics lessons made us realize our mistakes/…I corrected my mistakes more easily.*	2	2
	Geogebra software has a dynamic structure / shapes	*...we moved their sides/...we observed whether the angles changed by moving them/...It is better when we draw. It allows us to drag/…I do not have to draw in my exercise book over and over again.*	3	2

According to Table 7, it was perceived that in the learning environment created by using DGS, there is not any difference in the opinions of students, who have analytical and holistic thinking styles about teaching polygonal subject. Students of both thinking styles; cognitive, emotional, and software programs, they were using similar expressions in three separate themes. It was understood from the diaries of students with an analytical or holistic thinking style that this learning environment has an equivalent effect on both thinking styles. It indicates that the students have positive opinions regarding the learning environment.

Discussion, Conclusion and Suggestions

The present study initially aimed to determine the impact of a learning environment assisted with DGS on the SOLO taxonomy levels of students with analytical and holistic thinking styles. The pre-structural, uni-structural and multi-structural levels of the SOLO taxonomy reflect quantitative learning, while the higher levels indicate qualitative learning. When the SOLO taxonomy was used to evaluate the quantitative and qualitative complexity (Leung, 2000) of students' responses, it was revealed that there were transitions to multi-dimensional structures in the Final-DTS responses of students having an analytical thinking style in a learning environment assisted with DGS, that this kind of a learning environment enhanced the quantitative aspect of the structural complexity in terms of concepts related to polygons, yet students experienced difficulties in combining all the information they had within a consistent structure. In other words, in the responses of students with this kind of a thinking style, it was recognized that they took into consideration numerous features related to the question but experienced difficulty in establishing a relationship among them and in establishing generalizations.

As a result of the evaluation of the quantitative and qualitative complexity in students' responses by using the SOLO taxonomy, it was revealed that a learning environment assisted with DGS enhanced the quantitive aspect of structural complexity in the responses of students with a holistic thinking style. While detailed student responses, that is responses reflecting the quantitative aspect of structural complexity, are encountered in this kind of a learning environment, there are very few responses that reflect the qualitative aspect of structural complexity, which refers to how well the student can establish a relationship among the details mentioned in the responses. That is, in the responses of these kinds of students, two or three features are addressed independent of each other and it is observed that students have difficulty in combining and supporting these features with strong proof.

When the learning environment created using DGS was assessed by the SOLO taxonomy, it could be explained that students who have both analytical and holistic thinking styles have contributed to the quantitative direction of structural complexity in learning and made progress in thinking level as follows. Concepts were visualized using DGS, which enables the polygons to be explored with concrete representations. Through these concrete presentations, students have the opportunity to think about the relationship among polygons. It was thought that this learning environment contributes especially to the creation of new polygons with the same features and drawings of any polygon. Acccording to Köse (2008), the DGS environment gave benefits to students in terms of visualization and association. It also drew attention to the fact that students should be introduced to DGS-supported environments starting from primary education. Hohenwarter (2004) stated that Geogebra software is a versatile system for geometry education between the ages of 10-18, and stated that teachers can actively use this program in practice, in visualization and in the preparation of teaching materials. Accordingly, it indicated that dynamic softwares, which allow the creation of appropriate atmosphere when polygons are learned by concrete representations, should be included in the secondary school education process.

The learning environment in which DGS is used offers the possibility to easily model the activities and problems in the event for students with both analytical thinking style and holistic thinking style. In particular, the answers given by the students in the FLAE application, where each student was allowed to use a computer, were examined after the assessment implementation and it was seen that they used the building features of dynamic software in answering the questions. This is thought to be the effect both styles of thinking have on reaching higher levels of answers in the SOLO taxonomy. In a study by Christou, Mousoulides, Pittalis, and Pitta-Pantazi (2005), Dynamic geometry software supports students in hypothesizing, experiencing, generalizing, modeling the steps in question and solving problems. Moreover, the software has a role that facilitates problem solving and encourages cognitive processes used effectively. Students who used the Geogebra software actively in the study noticed during their practice that learning was carried out in their own way and with their own strategies. Accordingly, it is recommended that such well-structured learning environments, which facilitate the solution of each problem to students with different thinking styles, be designed. It is believed that the use of this software can be beneficial in other sub-learning domains of mathematics.

In order to reach higher level answers in student thinking levels, it is thought that it is effective for the students to apply alternative thinking options, to be able to show active participation in the lesson and to share their ideas with group members and to offer the opportunity to structure their knowledge. Tutak and Birgin (2008) point out that the dynamic geometry software-assisted environment supports students' participation in the process and helps them to make experiments and test their knowledge in this learning environment. It was stated that students learn to learn mathematics individually or in groups, and that this environment possesses desirable qualities (Hohenwarter & Fuchs, 2004). In addition, individuals with analytical and holistic thinking style are more likely to be successful in the tasks they undertake together. This is because they build a structure that supports each other in terms of complementing the different aspects of the task they are working on and without looking at the missing parts (Sternberg, 1997). This means that dynamic learning environments should be included in the schools as they allow individuals with different thinking styles to interact with each other in the learning process and support their participation in the process.

Some of students did not achieve a sufficient increase in the level of SOLO taxonomy. There were not many answers reflecting the qualitative direction of the structural complexity in the learning environment, indicating that there were not enough responses at the relational level of the SOLO taxonomy levels in student responses in FLAE, may be due to their perception of this environment as a game (Memişoğlu, 2005; Musan, 2012). Furthermore, students' familiarity with a new learning environment and adaptation of this process to the new learning environment can be seen as a reason for the students to have fewer qualitative responses to the SOLO taxonomy levels. In this sense, an investigation of the effect of the long-term outcomes of the study by ensuring that students interact with such learning environments in certain intervals is recommended.

Secondly, in this study, it was also aimed to determine how the students with either the analytical or holistic thinking styles learn in the learning environment assisted with DGS, how the differentiation of the SOLO taxonomy levels before and after the course occurred, and how the implementation took place. Accordingly, in the PLAE applied before teaching in the dynamic environment, student responses in both thinking styles are predominantly at the uni-structural (US) level of the SOLO taxonomy. This indicates that there is no significant difference among the readiness levels of 7th grade students with analytical or holistic thinking styles, and that they are at similar stages of the SOLO taxonomy in terms of preliminary knowledge about the subject.

According to the SOLO taxonomy analysis of student responses in the FLAE following the activities performed in the learning environment created by using DGS, information was obtained that they are mainly at the uni-structural (US) and multi-structural (MS) levels in the thinking stages. According to this finding, it was determined that the learning environment assisted with DGS did not bring about a differentiation between the two thinking styles although the increase in the SOLO levels in terms of the students with analytical and holistic thinking style was brought about. It could not be stated that ones' thinking style is superior to the other or that it concentrates on a higher level. Therefore, individuals with different thinking styles did not have any superiority over the other in the SOLO taxonomy levels; the learning environment assisted with DGS also shows that different thinking styles do not provide any advantage over the other. Furthermore, this study shows that the students receiving education in a learning environment assisted with DGS do not change their SOLO taxonomy levels according to their thinking style. The reason for this is that a dynamic environment assisted with DGS is conducive to both individuals with a tendency of analytical thinking who parse knowledge and operate it sequentially and those who have holistic thinking styles pursuing their obtained answers in similar activities as resources and producing their own methods instead of thinking intuitively or using a step-by-step approach. Kaya (2009) investigated students' (grades 6, 7, 8) thinking styles according to academic achievement and found that mathematics success did not differ according to holistic or detailed style of thniking. He explains that mathematics success does not vary according to thinking style, and the learning environment in schools do not support these styles. However, there is no information about whether the course is conducted with the study group with whom the research is conducted or whether it is a learning environment where a dynamic software is used. Umay and Arıol (2011) stated that there is no significant difference between the groups with dominant analytical thinking styles and dominant analytical thinking styles in terms of problem solving performances in the study of holistic and analytical thinking styles in solving mathematical problems and examining the effect on selected solutions. This interpreted the fact that having analytical or holistic thinking styles did not provide advantages or disadvantages in solving mathematical problems. Researchers have come to the conclusion that each individual's innate style and low level of performance is not a factor in developing a problem-solving approach in accordance with the style of thinking later acquired by environmental ethics. This study also concluded that students who were learning in the learning environment using DGS did not differ in their thinking styles, SOLO taxonomy levels, and style performance or performance was not a factor.

This study also identified student views on the teaching of polygons in a learning environment assisted with DGS. It is understood that students who have both analytical and holistic thinking styles examine the expressions under three themes related to the cognitive, emotional and software programs, and they have similar opinions about the teaching of polygons in environments assisted with DGS. There is no significant difference between the analytical thinking style and the holistic thinking style in terms of the diversity of the expressions used in the diary. Because this designed dynamic environment allows students to experiment, each student has the opportunity to learn by experimenting with his own thinking style, and the fact that it is not a traditional learning environment allows them to act cognitively according to their own style. The DGS environment, which offers a learning process in which everyone can configure their knowledge according to their own thinking style, has come to the conclusion that the individuals in both styles overlap with their preferences. Research on defining possible different learning environments in which each student can take action based on one's own thinking style can contribute to the literature.

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