



## A Different Look at the Reasoning Process of Prospective Middle School Mathematics Teachers: Global Argumentation Structures \*

Özlem Erkek <sup>1</sup>, Mine Işıksal Bostan <sup>2</sup>

### Abstract

Argumentation is an important part of education in different disciplines such as science teaching and psychology. Numerous research studies have revealed that the use of argumentation practices in courses have a positive impact on students' both cognitive (critical thinking, reasoning, etc.) and affective abilities. Recently, the number of studies on argumentation in the field of mathematics education has increased. It is expected that prospective teachers today are key elements of the argumentation process as they can encourage students to produce strong mathematical arguments; hence, they themselves are also expected to develop their own argumentation skills. In the literature, there is a need for studies on the reasoning processes and argumentation structures of prospective teachers. Thus, the present study aimed to investigate the global argumentation structures used by prospective middle school mathematics teachers while they were solving geometry tasks. In line with this purpose, a qualitative case study was conducted with 8 prospective teachers who were enrolled in the elementary mathematics education department of a state university located in Ankara. Two argumentation applications were conducted with the participants. In the first application, two problems on triangles and in the second one two problems on circles were solved by the participants. Then, the global argumentation structures of the participants were examined. The results showed that mathematical reasoning of prospective middle school mathematics teachers was weak and, therefore, they mostly used simple global argumentation structures. On the other hand, it was found that the researcher's directions lead participants to use more complex global argumentation structures. Therefore, it was concluded that the teachers who orchestrate the argumentation process should have sufficient knowledge and experience regarding argumentation. This result revealed the importance of focusing on the development of mathematical reasoning skills of prospective teachers. Based on this, it is recommended that necessary adjustments be made in teacher training programs to include argumentation so that prospective teachers can be equipped with the ability to orchestrate argumentation in their lessons.

### Keywords

Argumentation  
Geometry  
Global argumentation  
Prospective middle school  
mathematics teachers  
Toulmin model

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## Introduction

Argumentation theory seeks to explain that reasoning is essentially a social ability (Mercier & Sperber, 2011). When individuals share their arguments in an environment of social interaction, both individuals presenting an argument and those evaluating / rebutting it benefit from the process. Argumentation theory is based on the view that *reasoning* is a cognitive talent that assists those presenting arguments while seeking reasons for their own opinions, and the listeners while evaluating these arguments (Mercier, 2011). Thus, it can be deduced that reasoning is essentially a social means for the argumentation process. In fact, there are even views in the literature advocating that reasoning is developed to cater to arguments. Considering that in studies based on argumentation individuals are in the process of social interaction and present their claims by continuously reasoning, it can be inferred that the concepts of reasoning and argumentation are intertwined and closely related to each other (Conner, Singletary, Smith, Wagner, & Francisco, 2014).

In the related literature, argumentation and argument emerge as different concepts. Argumentation is defined as the process of social interaction in which claims are constructed and evaluated individually or collaboratively in the light of experiential or conceptual proof, warrants, rebuttals and alternative views (Jiménez-Aleixandre & Erduran, 2008). According to another definition, argumentation is the total of the statements embedded with rhetorical discourse aiming to convince people of the accuracy or inaccuracy of an opinion (Antonini & Martignone, 2011). Hence, argumentation refers to a process. Argument, on the other hand, refers to the conclusion stated as a result of argumentation (Erkek, 2017). As opposed to traditional teaching methods by which knowledge is transferred from the teacher to the student, the argumentation process activates students and creates the opportunity for social learning. Students discuss their opinions freely during the argumentation process, present convincing arguments, rebutte their peers' arguments and reflect on and support their own opinions (Krummheuer, 2000; Stein, Engle, Smith, & Hughes, 2008). According to Inagaki, Hatano, and Morita (1998), such processes as student discussions, students' asking questions to their peers, peer evaluation of opinions, and direct peer feedback are effective in the construction of knowledge. Similarly, Yackel and Cobb (1996) have also advocated that interpretation of knowledge obtained as a result of peer interactions contribute to learning.

Numerous research studies in the literature highlight the positive impact of social interaction and argumentation on achievement (Kosko, Rougee, & Herbst, 2014; Walter & Barros, 2011; Yackel & Cobb, 1996). To illustrate, a study by Walter and Barros (2011) revealed that there was progress in the higher order mathematical thinking skills (Kosko et al., 2014) and reflective mathematical thinking skills of the students who worked on developing arguments and tried to seek different methods of solution to come to an agreement. Similarly, Yackel and Cobb (1996) emphasized the importance of social learning environment with respect to achievement in mathematics. Walter and Barros (2011) suggest that students need to be provided with these kinds of learning environments in which they can actively produce arguments and examine different methods of solution so that they can develop their reasoning and higher order mathematical thinking skills (Kosko et al., 2014). Likewise, it has been maintained in the literature that students' higher order reasoning, critical thinking and decision making skills can be developed by means of argumentation practices (Driver, Newton, & Osborne, 2000; Güzel, Erduran, & Ardaç, 2009). Hence, argumentation can be considered a beneficial process for the development of higher order thinking skills of both teachers and students. The contribution that the present study will make to the literature by evaluating the competencies of prospective teachers - the to-be teachers - in argumentation is of significance as these teachers will be directing the argumentation process, which has a significant impact on students' learning processes and the development of their reasoning and higher order thinking skills. Accordingly, the present study aimed to evaluate prospective teachers' reasoning skills during their argumentation by examining the global argumentation structures they utilized.

### *Significance of the Study*

Studies in the literature highlight the importance of the teacher's role with respect to the quality of the mathematics addressed in class (Heinze & Reiss, 2007; Yackel & Cobb, 1996). Teachers are key elements that enable students to construct strong mathematical arguments, direct the argumentation process, and establish the social environment where students interact with each other (Rice, 2012). At this point, teachers are expected to possess certain skills so that a productive argumentation process can be established. Researchers advocate that teachers should be able to construct strong mathematical arguments and be able to provide proof so that they can evaluate students' explanations and arguments in class (Rice, 2012). However, no sufficient information at the national level regarding the impact of the argumentation process on learning mathematics can be encountered in the accessible literature since the field of mathematics education in Turkey lacks sufficient research. It is for this reason that examining the argumentation structures utilized by prospective teachers can shed light on the extent to which they can engage in mathematical reasoning and how they will structure the lessons they will do in their own classes in the future.

In the undergraduate courses in the science teacher education programs in Turkey, argumentation-based science courses and applications are frequently encountered, and the topic of argumentation is addressed in many of the lessons within the teacher education process (YÖK, 2018). Moreover, there are numerous scientific studies on argumentation-based science education (Kıngır, Geban, & Günel, 2013; Özdem, Ertepinar, Çakıroğlu, & Erduran, 2013). On the other hand, an examination of the undergraduate programs for elementary mathematics teacher education in Turkey reveals that argumentation is not yet actively practised; it does not sufficiently take place in either must or elective courses. In fact, it can be claimed that many of these prospective mathematics teachers graduate from university without experiencing the argumentation process (Erkek, 2017). Researchers assert that if teachers are to direct the argumentation process effectively, it will be beneficial for them to attend, as students, classes where argumentation is implemented (Prusak, Hershkowitz, & Schwarz, 2012). For this reason, prior to the onset of their teaching profession, prospective teachers' participation in an argument construction process as a student is believed to be beneficial so that they can establish such an environment in their future classes. Furthermore, it is of utmost importance that the argumentation structures employed by prospective teachers be examined so that middle school mathematics teachers who can implement argumentation-based lesson can be trained and that deficiencies in teacher education programs can be identified.

Thus, the present study aimed to raise awareness towards updating mathematics education undergraduate programs in teacher education institutions so that they include argumentation practices and to provide literature with preliminary information for future studies on argumentation in the field of mathematics education. Based on the studies in the literature, the present study aimed to examine the global argumentation structures that prospective middle school mathematics teachers developed while solving geometry tasks. To this end, the study sought the response to the following research question: 'What argumentation structures do prospective middle school mathematics teachers present while solving geometry tasks?'

### *Argument Model (Toulmin)*

In the literature, the argumentation studies spread initially on the basis of the argument model and components proposed in Toulmin's (1958) book 'The uses of Argument'. According to Toulmin's (1958) argument model, which is frequently referred to in research, an argument is composed of three basic components: *claim*, *data* and *warrant*. In addition to these basic components, the arguments may also include *backing*, *qualifier* and *rebuttal*, which are the three auxiliary components. *Claim* is defined as "conclusion of the argument" (Toulmin, 1958, p. 101). *Data* refers to "facts we appeal to as the foundation of the claim, or minor premise" (Toulmin, 1958, p. 101). *Warrant* is defined as "The statement authorising the move from the data to the claim, or major premise" (Toulmin, 1958, p. 101). The auxiliary components *backing* refers to "Further reason to believe the warrant" (Toulmin, 1958, p. 101); *qualifier* to "The statements which express the force of the claim" (Toulmin, 1958, p. 101) while *rebuttal* to "The

exceptional conditions which might be capable of defeating or rebutting the warranted conclusion” (Toulmin, 1958, p. 101). In the Toulmin’s (1958) argument model the relationships between these components were summarized as in Figure 1.

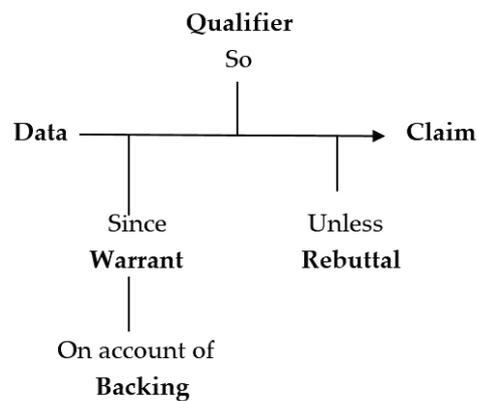


Figure 1. Toulmin’s Argument Model

In order to make the argument model given in Figure 1 more understandable, the sample argument diagram given by Toulmin (1958) in his book is presented in Figure 2.

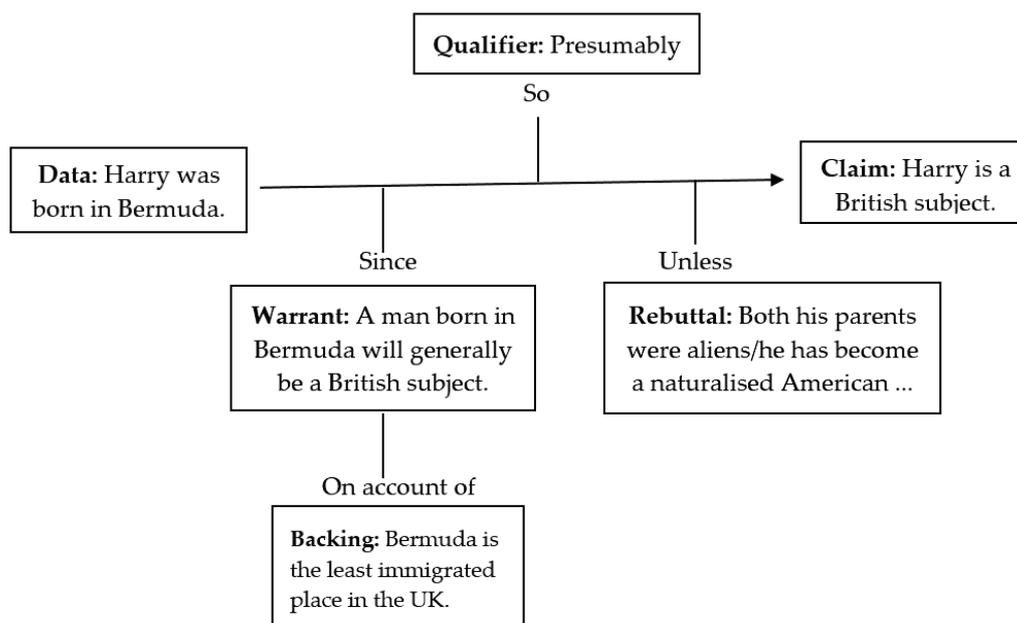


Figure 2. Sample Argument (Toulmin, 1958, p. 94)

Since Toulmin's argument model was independent of the field it was used in many different fields such as science, philosophy and mathematics. The model, first used in mathematics by Krummheuer (1995), allowed many researchers to easily analyze individual or collective argumentation processes (Hollebrands, Conner, & Smith, 2010; Inglis, Mejia-Ramos, & Simpson, 2007; Giannakoulis, Mastorides, Potari, & Zachariades, 2010; Krummheuer, 2007; Moore-Russo, Conner, & Rugg, 2011; Pedemonte, 2007; Pedemonte & Reid, 2011; Yackel, 2001; Nardi, Biza, & Zachariades, 2012). In this study, it was aimed to examine the argumentation process structures in the class discussions and the pair-works. For this purpose, it was decided to use the argument model developed by Toulmin for the presentation of the arguments.

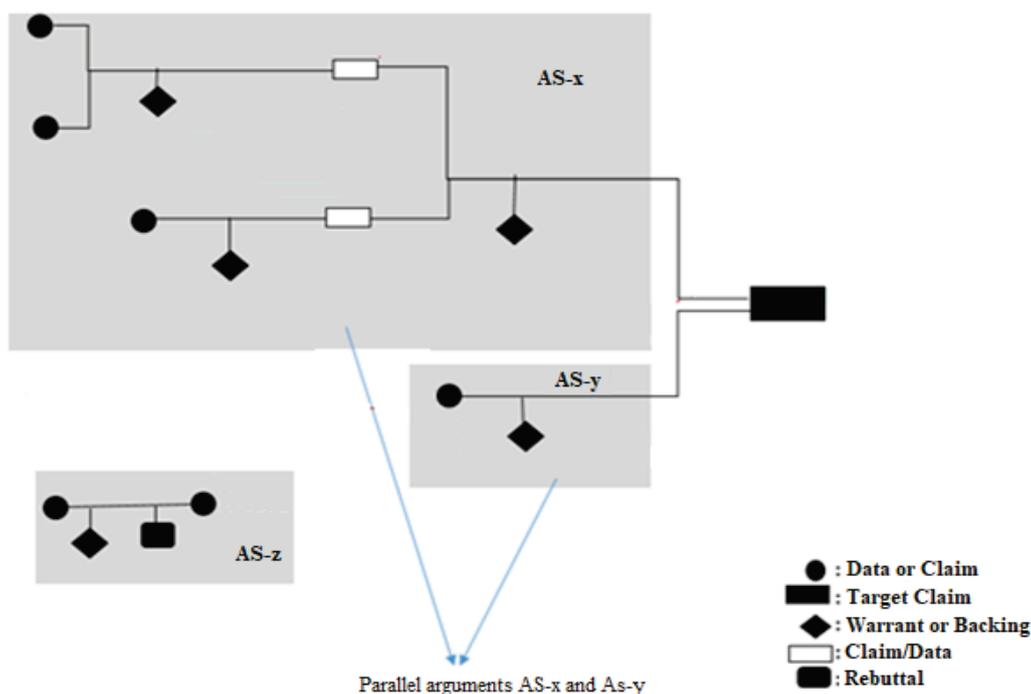
### ***Structures of Argumentation***

In studies on argumentation in the literature, some researchers are found to have focused on the different types of arguments (Aberdein, 2005; Viholainen, 2011), while others are observed to have dwelt on the warrant component of arguments (Inglis et al., 2007; Nardi et al., 2012; Walter & Barros, 2011). On the other hand, some researchers suggest that structures indicating the interrelationships of arguments be examined in order to examine the argumentation process more comprehensively and to draw conclusions (Knipping, 2008; Walter & Barros, 2011). In a study conducted on proof Knipping (2008) aimed to develop the argumentation model of Toulmin (Reid & Knipping, 2010), and hence, examined the global and local argumentation structures employed by students. Knipping (2008) advocates that examining argumentation structures is beneficial to understand the complexities inherent in teaching and in providing proof. Local argumentation analysis, which refers to the examination of warrant underlying each argument separately (Knipping, 2008), is out of the scope of the present study. The global argumentation structure analysis, proposed by Knipping (2008), forms the basis of the present study.

Global argumentation structure analysis is defined as the anatomical analysis of the structure of interrelated arguments (Knipping, 2008). Some of the claims made in argumentations presented during argumentation can be used as data in arguments put forward at later stages and, thus, interrelated argumentation streams [AS] are formed (Reid & Knipping, 2010). According to Reid and Knipping (2010), these argumentation streams do not generally progress in a linear fashion and, thus, argumentation structures become more complex and difficult to analyze. Based on this condition, Knipping (2008) proposed an analysis method (Global Argumentation Analysis) to facilitate researchers' analyses of complex relationships among arguments and suggested that more research be carried out on the analysis of global argumentation structures.

### ***Global Argumentation***

The theoretical framework developed by Knipping (2008) and Reid and Knipping (2010) was utilized in the present study to identify the global argumentation structures employed by prospective middle school mathematics teachers. Within this framework, the in-class argumentation processes are categorized by mapping them onto a web diagram and these general structures are referred to as global argumentation structure. Four types of global argumentation structures are mentioned in the literature: *source-structure*, *reservoir-structure*, *spiral-structure* and *gathering-structure*. Sample diagrams for these structures are presented in studies conducted by Knipping (2008) and Reid and Knipping (2010). To gain a deeper insight into the properties of these structures, it will be of benefit to define the concepts of argumentation steps, argumentation stream, and parallel arguments. In the literature, each statement that entails the components of data, claim and warrant is referred to as argument/argumentation step (Knipping, 2008). As for argumentation stream, it is defined as a series of argumentation steps that warrant/support the target claim (Knipping, 2008). Thus, in the present study, argumentation stream (AS) is defined as the structure formed of interrelated argumentation steps supporting the target claim. Finally, parallel arguments are defined as different arguments supporting the same conclusion (claim) within the argumentation stream (Knipping, 2008). These types of arguments emerge when different arguments are produced for the same conclusion or when different warrants are presented. One or all of the parallel arguments can be in the form of argument stream as well. In other words, argumentation streams that are parallel to each other can also be encountered. Knipping (2008) proposed a schematic diagram for global argumentation structures. For an explicit presentation of these mentioned definitions, a sample argumentation stream diagram, proposed by Knipping (2008), is displayed in Figure 3 below.



**Figure 3.** Sample Argumentation Stream and Its Model

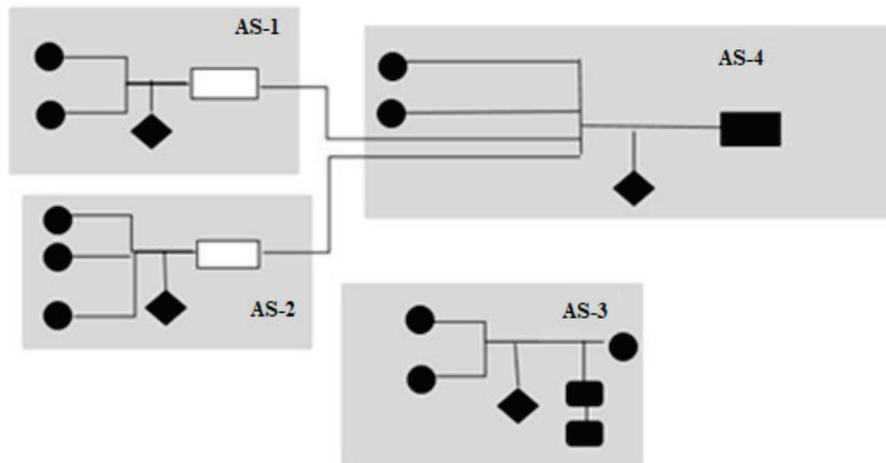
A simple argumentation stream can be observed in Figure 3. Each of the components indicated with big grey quadrilaterals depicts an argumentation stream (AS). Each component consisting of data, claim and warrant (rebuttal) is an argument. As for the argumentation streams in Figure 3, which arrive at the same target conclusion by means of different methods of solution, they depict the parallel argumentation streams indicated as AA-x and AA-y. AA-z is a rebutted argument; at the same time, it is an argumentation stream that is disconnected to the main structure.

During in-class argumentation, arguments can be interrelated, a piece of information that functions as a claim in one argument can be used as data in another argument, more than one datum or warrant can be presented in an argument, and sometimes intermediate claims can even emerge during interim steps prior to reaching the main claim (target conclusion). To visually examine these complex arguments, it is essential to form the global argumentations structures. Examining global argumentation structures enables researchers to see the big picture of the argumentation taking place in class. If individuals are guided effectively during the argumentation process, they can explicitly discuss the conceptual relationships in their minds and the misconceptions, if any. In the literature, Walter and Barros (2011), who examined argumentation structures, advocate that by examining the structures of arguments developed while students are carrying out problem-solving tasks, one can gain important information regarding students' comprehension processes of the problem statement and their reasoning skills. Hence, it can be concluded that by examining the argumentation structures of the prospective teachers, who will be guiding the argumentation process in their future classes, important information can be obtained as regards their mathematical thinking and reasoning skills (Kosko et al., 2014; Walter & Barros, 2011). In conclusion, in the present study, by examining the global argumentation structures of prospective middle school teachers, information regarding their mathematical thinking and reasoning skills was obtained.

#### ***Global Argumentation Structures***

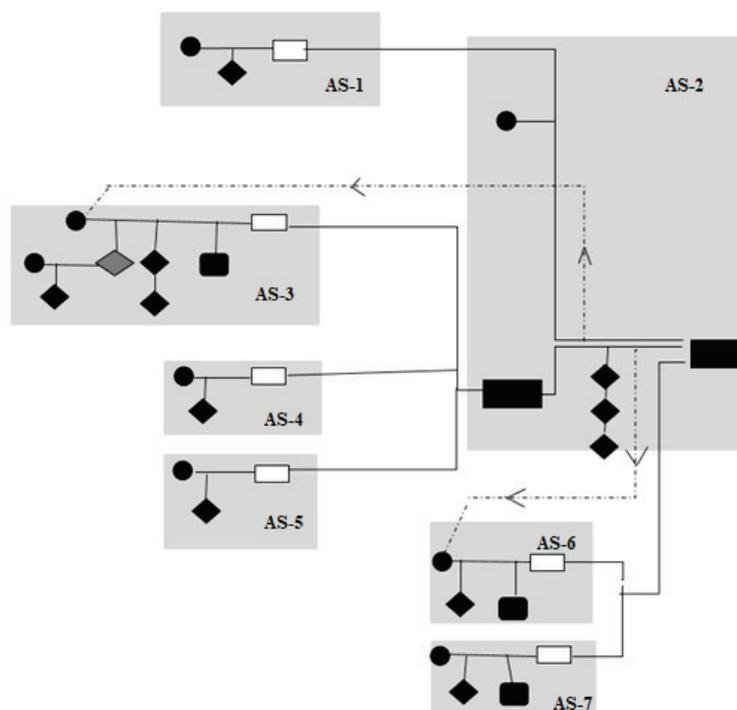
*Source-structure:* In this structure, opinions and arguments are presented based on different sources. Reid and Knipping (2010) have explained this structure by resembling it to the unification of headwaters streaming out of different sources to form rivers. Other distinguishing features of this structure: the presence of argumentation streams that have no connection to the main structure (AS-3

in Figure 4), the presence of parallel arguments supporting the same claim (AS-1 and AS-2 in Figure 4), the presence of argumentation steps with more than one datum (AS-1, AS-2 and AS-3 in Figure 4) and the existence of rebuttal components (AS-3 in Figure 4) (Reid & Knipping, 2010). In addition to these, diagrams roughly resembling the figure of a funnel, defined as the funneling effect, in which many arguments are evaluated and combined to reach the target conclusion, are evidently observed in this structure (Reid & Knipping, 2010). As can be observed in Figure 4, in some situations, the component that functions as the claim in the first argument can be used as the data in a subsequent argument. Knipping (2008) portrayed these kinds of claim/data components as hollow rectangles (see Figure 4, AS-1 and AS-2).



**Figure 4.** Sample Model of Source-Structure Obtained in this Study

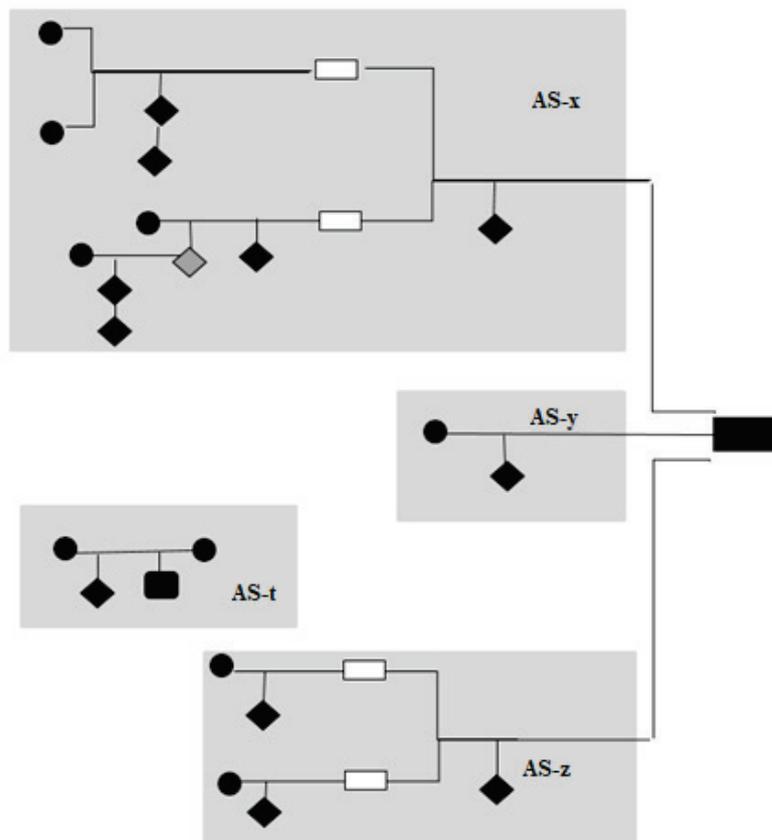
*Reservoir-structure:* In this structure, as defined by Knipping (2008), there are intermediate conclusions/claims which are independently self-sufficient. These intermediate claims are resembled to reservoirs that retain and purify water before they pass onto the next stage (Reid & Knipping, 2010). Figure 5 displays a sample reservoir-structure obtained in the present study by means of a diagram.



**Figure 5.** Sample Model of Reservoir-Structure Obtained in this Study

The most important distinguishing property of the reservoir-structure is that it occasionally goes backward and then continues forward during the discussion phase of reasoning with the aim of supporting the arguments with more data. In Figure 5, the dashed lines, the direction of which are shown with arrows, shows the stages at which reasoning went backwards. AS-1 is followed by AS-2, but before arriving at the target conclusion, more detailed inferences are drawn by proceeding to AS-3. Similarly, there is a backward move while proceeding towards the target conclusion with AS-2 after AS-4 and AS-5, and then by making a forward inference with AS-6, a move has been made to AS-7 and finally, by moving onto AS-2, the target conclusion is reached. The target conclusion is reached after sufficient support is provided with future inferences (Reid & Knipping, 2010). When compared to the other structures, the process in this structure entails more in-depth argumentation because the participants evaluate the arguments repeatedly and return to previous arguments to make additional explanations.

*Spiral-structure:* Even though the properties of spiral-structure resemble those of source-structure, Reid and Knipping (2010) stated that these two structures can be distinguished by examining the location of parallel arguments in the diagram. In source-structure, parallel arguments are located at the beginning of the argumentation process, while in spiral-structure, parallel argumentation streams are located at the very end of the argumentation (Reid and Knipping, 2010). Thus, while the conclusion arrived at in the parallel argument is the 'claim/data' component in the source-structure, it is the 'target conclusion' component in the spiral-structure. A sample diagram of spiral-structure obtained in the present study is portrayed in Figure 6.

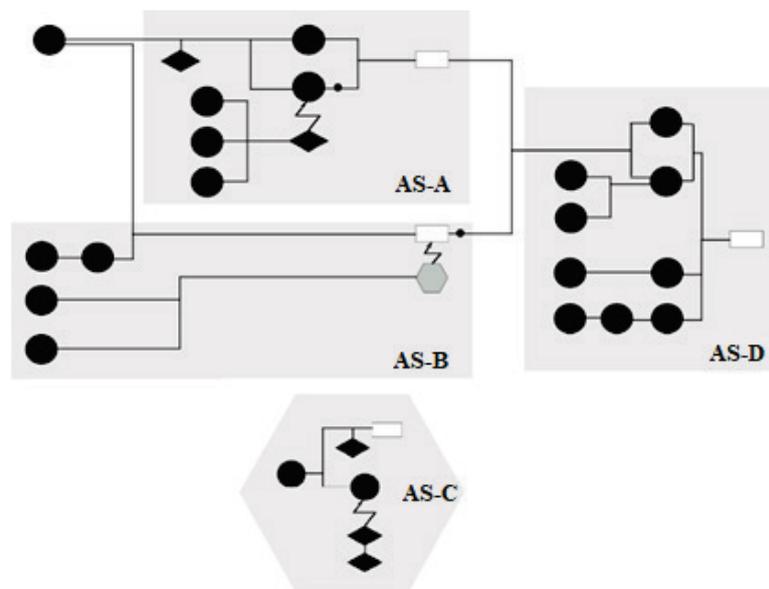


**Figure 6.** Sample Model of Spiral-Structure Obtained in this Study

Parallel structures, which are characteristic of spiral-structure, are displayed in Figure 6 as AS-x, AS-y, and AS-z. Each one of these arguments arrive at a target conclusion depicted as a black rectangle in the diagram and form images parallel to each other. As for AS-t, it is displayed separately as it is

independent of the other arguments and is an argument that does not reach a conclusion. As can be observed in all the structures, including the spiral-structure, there are arguments like AS-t that are disconnected to the main structure. These arguments do not distort the overall spiral-structure property.

*Gathering-structure:* In this structure, a lot of data are collected to support some interrelated conclusions/claims. New data are added to the argument streams as the argument proceeds. In other words, not all the data are mentioned in the initial stage; new data emerge as the argument proceeds. In the gathering-structure, there are no parallel arguments and argument streams proceed independent of the the main structure. These properties distinguish gathering-structure from source-structure and spiral-structure. Moreover, in the gathering-structure, there are no back and forth processes of reasoning as observed in the reservoir-structure. Figure 7 depicts a sample diagram of gathering-data from the study of Reid and Knipping (2010).



**Figure 7.** Sample Model of Gathering-Structure Taken from Reid ve Knipping's (2010, p. 190) Study

It is evident that quite a lot of data are used, that is collected, in the argument streams (AS-A, AS-B and AS-D) in the sample diagram portrayed in Figure 7. In addition, the presence of arguments for which no warrant is clearly stated can be observed. The hexagonal figure in argument stream-B (AS-B) represents a rebuttal, and a detail of this rebuttal is shown in argument stream-C (AS-C). When some of the argument components are more than one in this structure, they display a collective image. The collective image is a property that distinguishes the gathering-structure from the other structures.

## Method

In the present study the case study design, one of the qualitative research methods, was utilized. The case study is defined as the explanation and analysis of a confined phenomenon such as a program, institution, process or social unit in an intensive and holistic way (Merriam, 2009). The case study was considered an appropriate research method to examine the argumentation process of the participants was to be examined.

The implementation is limited to the process of argumentation in which prospective teachers solve four geometry tasks. In this regard, an in-depth examination was made of the global argumentation structures produced by prospective middle school mathematics teachers in the mentioned environment.

### *Sample (Participants)*

The sample group was determined subsequent to certain stages. Initially, a pilot study was conducted with six volunteer prospective teachers to decide upon the context of implementation and

the geometry problems to be implemented. The pilot study aimed to identify the geometry tasks upon which the arguments were to be based, to determine the time span needed for each implementation and the number of geometry tasks that could be solved in these implementations, to evaluate whether or not the statements in the geometry tasks were explicit and clear, to identify the challenges that could be experienced during the implementation stage, and to decide on the number of people in each group for an effective implementation of argumentation. In the first implementation of the pilot study, four geometry tasks were prepared. However, it was observed that the participants got tired while discussing the last two geometry tasks, they found it difficult to produce arguments and, hence, the argumentation process did not proceed effectively. Therefore, it was concluded that in the main implementation, two geometry tasks would be sufficient in a single implementation. On the other hand, in the pilot study, the researcher worked with groups of twos and threes in order to decide on the number of people to form the small groups in the main study. In some cases, it was observed in the groups of threes that one member remained quiet, while the other two members took part in the discussion. Thus, it was decided that the small groups in the main study would consist of two members. According to the results of the video analysis, it was observed that when sufficient time for pair discussions was not allocated, the participants did not participate in class argumentation very much as they were still in the process of thinking about their own solutions. Hence, the decision was taken to allocate more time to the pair group discussions in the main study. According to another important outcome of the pilot study was that when there was a time span between the implementation and the post-interviews, it was realized that the participants forgot some of the details regarding the argumentation process. Therefore, it was decided that the interviews were to be held a day after the implementation and that the video recording of the implementation would be shown quickly to the participant prior to the interview so that the details could be remembered. Finally, as the geometry tasks prepared enabled the participants to use different methods of solution to produce numerous arguments, the content of each task (triangle and circle) was not changed.

By taking into consideration the results of the pilot study, purposive sampling method was used to select 8 prospective middle school mathematics teachers from among senior students in the primary mathematics teacher education department in a state university in Ankara for the main implementation. For a productive implementation of argumentation, participants who were not shy, but talkative, motivated, and outspoken, and were able to express criticisms and present arguments were selected for the present study. Table 1 presents information regarding pseudonym, gender, and year level of the participants working in pairs.

**Table 1.** Information Related to the Participants

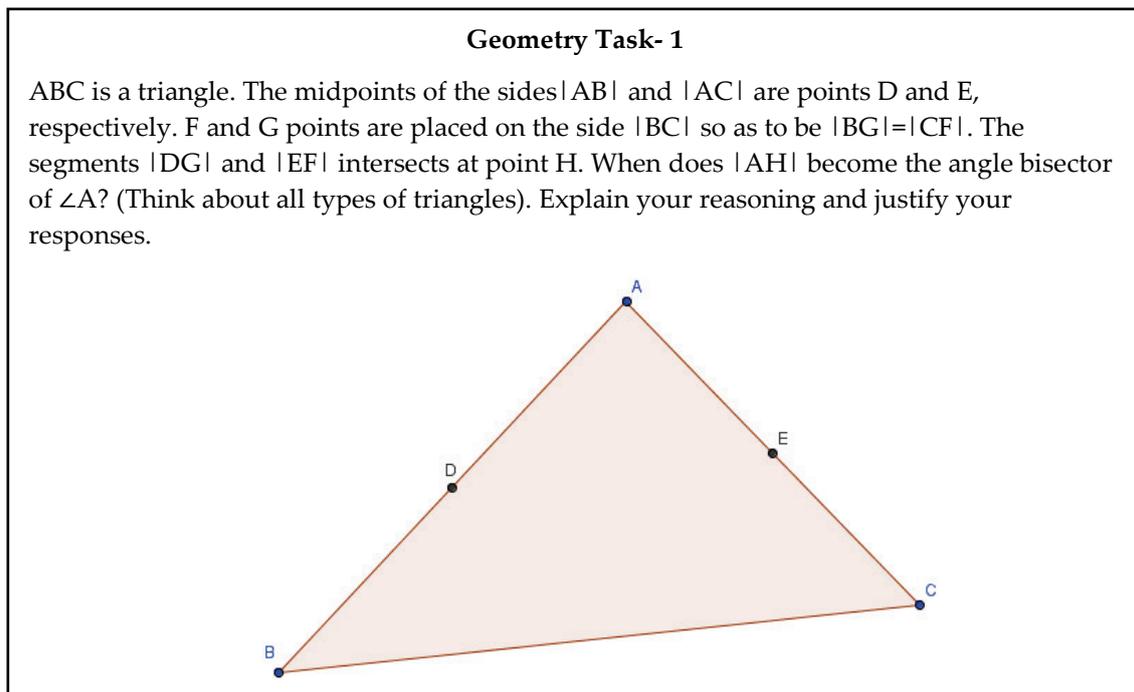
	<b>Pseudonym</b>	<b>Gender</b>	<b>Grade</b>
Group 1	Beren	Female	4
	Özer	Male	4
Group 2	Gözde	Female	4
	Umut	Female	4
Group 3	Bahar	Female	4
	Güler	Female	4
Group 4	İnci	Female	4
	Pelin	Female	4

#### ***Data Collection Tools***

The data collection tools of the study were the inventory consisting of geometry tasks and the interview questions. In the present study, four geometry tasks (GT1, GT2, GT3, GT4) were used: two on the topic of triangles and two on the topic of circles. Despite the presence of case studies using one task in literature, it was decided that the present study be based on more than one geometry task for the sake of revealing tendencies regarding the global argumentation structures used by prospective teachers

while solving geometry tasks. Within the scope of the study, the topics of triangles and circles were chosen, and two geometry tasks were prepared for each topic. The required approvals to use these geometry tasks were received and subsequently, these tasks were adapted to fit the purpose of the study. Hence, for effective argumentation, the tasks were designed so as to encourage participants to develop arguments, and direct them to present multiple methods of solution and provide warrants. Entailing more than one method of solution supports participants' presentation of arguments from different perspectives. In addition, such statements as 'Justify your responses', 'Explain your reasons', and 'Explain why there is such a relationship' were added to the geometry tasks to encourage the participants to question their opinions and to present warrants. Details regarding the geometry tasks are presented below.

The first geometry task (GT1) was taken from Ceylan's (2012) master's thesis (see Figure 8). The solution of this task requires important skills in argumentation, such as constructing a hypothesis and testing assumptions. The task in the worksheet was presented as follows:



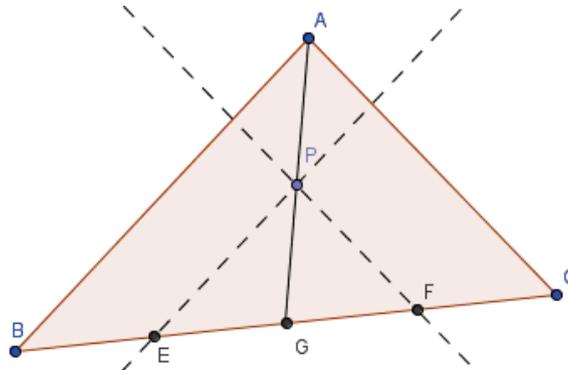
**Figure 8.** First Geometry Task (GT1)

The second geometry task (GT2) was taken from the PhD dissertation of Iranzo-Doménech (2009) and used after it was adapted (see Figure 9). This task was included in the study as it had a structure encouraging the development of arguments, but it was adapted to assume a structure encouraging the presentation of warrants to arguments as well.

### GeometryTask- 2

Let  $P$  be any point on the median of  $|AG|$  of a triangle  $ABC$ . Let  $m$  and  $n$  parallel lines through  $P$  to the sides  $|AB|$  and  $|AC|$  of the triangle.

1. What relation is there between the segments  $|EG|$  and  $|GF|$ ? Explain your reasoning.
2. What if the triangle  $ABC$  is equilateral or isosceles triangle? Can any generalization be made for the relation between the segments  $|EG|$  and  $|GF|$ . Explain your reasoning.



3. Where must be the point  $P$  positioned such that  $|BE|=|EF|=|FC|$ . What if the triangle  $ABC$  is equilateral or isosceles triangle? Justify your solution.

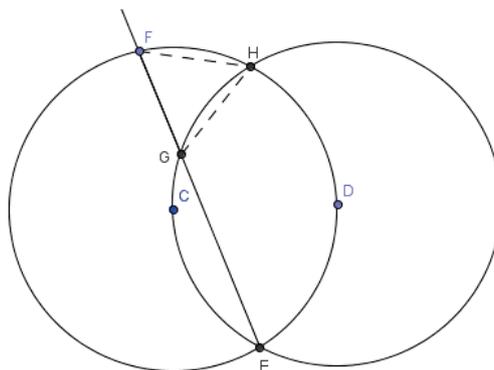
Figure 9. Second Geometry Task (GT2)

The third geometry task (see Figure 10) and the fourth geometry task (see Figure 11) were taken from a book titled 'Challenging Problems in Geometry', by Posamentier and Salkind (1988), but were adapted to assume a structure encouraging participants' presentation of warrants to arguments.

### Geometry Task- 3

Two circles each of which passes through the center of the other, with centers  $C$  and  $D$  were given. A line from  $E$  intersects circles at  $F$  and  $G$ .

1.  $|FG|=6$ , compute the area of the triangle  $FGH$ ? Justify your solution.



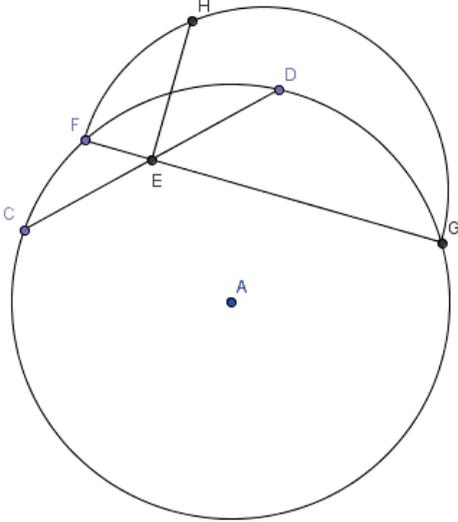
2. If  $r$  is the measure of the radius of each circle, find the minimum value and the maximum value of the area of triangle  $FGH$ . Justify your solution.

Figure 10. Third Geometry Task (GT3)

**Geometry Task- 4**

$|CD|$  is a chord of a circle, and bisected by another chord  $|FG|$  at point  $E$ . A semicircle is drawn with diameter  $|FG|$ .  $|EH|$ , perpendicular to  $|FG|$ , meets this semicircle at point  $H$ .

1. Prove that  $|EH|=|CE|$  and justify your reasoning.



2. Show whether the theorem is trivial if chord  $|FG|$  is a diameter of the first circle, or if  $|FG|$  coincides with  $|CD|$ . Justify your reasoning.

**Figure 11.** Fourth Geometry Task (GT4)

During the argumentation process, some argument components were not explicitly stated by the participants, and some parts were partially expressed. With aim of clarifying the incomplete conditions during the argumentation process, interviews were held after the implementations with a selected group of participants. Thus, the incomplete argument components in the argumentation process were revealed by enabling participants to make further explanations. To illustrate, while solving the fourth geometry task, one group did not discuss among themselves the second question (see Figure 11). Thus, in the interview, the researcher asked them to discuss the solution to this task and warrant their responses. Furthermore, unclear argument components were completed by asking the participants such questions as 'Here you say ... What do you base this on? Can you provide warrant for this? Is there anything that can refute your opinion? Do you think this is correct in all conditions?'

#### ***Data Collection Process***

One of the researchers carried out the implementation with eight participants outside of class hours in a classroom arranged for them. Each group in the implementation was given the geometry tasks, the details of which are outlined in the previous section, so that they could generate different methods of solution and provide warrants for their responses. Two implementations, which lasted approximately three hours each, were carried out on two separate days. In the first implementation, the triangle tasks, and in the second implementation, conducted in the following week, the circle tasks were solved by the participants. The implementations were recorded by means of camera and voice recording devices.

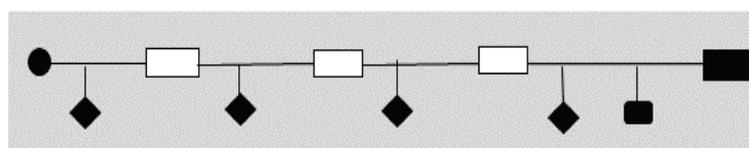
At the beginning of the implementation, each group was given one copy of the worksheet prepared for the first geometry task (GT1). Subsequently, the participants were asked to discuss by thinking aloud the task in pairs, as indicated in Table 1. During this process, the participants were encouraged to find different methods of solution, to continuously discuss their opinions with their group members during the group discussion, and with the other groups during the general argumentation, to follow the arguments in the implementation and evaluate their friends' opinions.

Meanwhile, the researcher walked around the classroom and observed the groups at work and responded to the questions asked by the participants without providing guidance. Moreover, the researcher enabled the participants to continue discussing by providing clues at points where they could not proceed. After the group discussions ended, starting from the volunteer groups, all the groups were asked to present their solutions to the rest of the class by holding class discussions. Each participant had the right to intervene in the solution and question any aspect of the solution during the class discussion. At this stage, to enable the participants to provide warrant for the arguments they presented, the researcher, at certain points, asked such questions as ‘Why did you do it in that way?’, ‘Can you elaborate on that part a little more?’, ‘What are you basing what you say on?’, and ‘Can you explain how you did this part?’ After all the groups shared their different solutions with the class, the researcher ended the argumentation process. After the first geometry task (GT1) was solved in this way, the worksheets distributed to the groups were collected, and the worksheet for the second geometry task (GT2) was given to each group so that they could solve the task in the same way. After the implementation was completed, the researcher chose a focus group whose video recordings of the groupwork and class discussions were watched. During this stage, the researcher identified the argument components (data, claim, warrant) that were not expressed in the arguments developed or were not sufficiently clarified and prepared interview questions to detail these components. On the following day of the implementation, an interview was held with the selected focus group and these interviews were also recorded by means of a video and voice recording devices. The implementation of the second week was carried out in the same manner; the participants were asked to solve GT3 and then GT4.

### *Data Analysis*

For data analysis, initially the arguments proposed by the participants during the argumentation process were to be identified. To identify these arguments and to establish a schematic diagram of these arguments, the argument model developed by Toulmin (1958) was employed. However, an inter-coder agreement study was conducted to identify the reliability and validity of the arguments and their components (data, claim, warrant). The coders were two expert researchers in the field of mathematics education. Initially, 25 % of the data were randomly selected and analyzed by both the coders and the researcher separately, thus identifying the arguments and their components. Subsequently, the researcher and the two coders came together to negotiate until they came to a consensus on all the arguments, and finally they determined the features of the argument components for the remaining analyses. Thus, the reliability study was completed. After the arguments were identified, the global argumentation structures of the participants were formed for each geometry task by using the global argumentation structures proposed by Knipping (2008) and Reid and Knipping (2010). A thick description of the arguments was made. In addition, observation recordings, interview recordings, and the notes taken by the participants during the implementation process were all analyzed in combination to establish triangulation. As a result, five different global argumentation structures were observed. Three of these were source-structure, reservoir-structure and spiral-structure, which were all explained comprehensively in the literature review section. The remaining two structures, line-structure and independent arguments-structure, do not exist in the literature, but appeared in the present study. Details regarding these new structures are explained below.

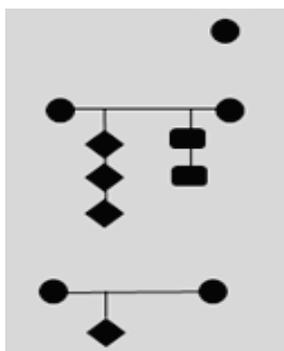
*Line-structure:* Some of the global structures obtained in the present study did not match with any of the structures within the theoretical framework. Therefore, the researchers created new names to these different structures that emerged during data analysis. One of these is the line-structure.



**Figure 12.** Sample Model of Line-Structure Observed in this Study

The distinguishing feature of this structure is that the arguments, as can be observed in Figure 12, are aligned in a linear fashion to arrive at the conclusion. The connection between the arguments are established through the claim/data components. One other property that distinguishes line-structure from source-structure and spiral-structure is the non-existence of parallel structures. Moreover, in line-structure, as opposed to the reservoir-structure, back and forth reasoning processes do not exist.

*Independent arguments-structure:* Another structure that emerged in the present study and does not exist within the theoretical framework is the independent arguments-structure. An example for this structure is displayed in Figure 13.



**Figure 13.** Sample Model of Independent Arguments-Structure Observed in this Study

Independent arguments-structure emerged when the participants could not solve the tasks but expressed opinions that came to their minds. In some situations, the participants who expressed opinions and then refuted them also constructed independent arguments. Apart from these, the participants who constructed an argument for each stage of the task and passed on also generated independent arguments. In conclusion, data analysis showed that disconnected arguments independent of each other were constructed. The researchers labelled this new structure as independent arguments-structure.

## Results

This section dwells on the findings which the data analyses yielded. The global argumentation structures that emerged during the argumentation process and the structures that were observed in each geometry task are summarized in Table 2.

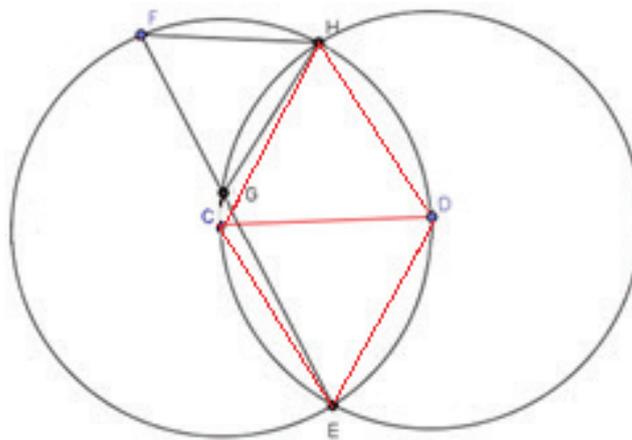
**Table 2.** Distribution of Global Argumentation Structures in Each Geometry Task

	<b>Global Argumentation Structures</b>
<b>Geometry Task 1 (GT1)</b>	1 Reservoir-Structure 1 Line-Structure 8 Independent Arguments-Structure
<b>Geometry Task 2 (GT2)</b>	1 Source-Structure 2 Spiral-Structure 2 Independent Arguments-Structure
<b>Geometry Task 3 (GT3)</b>	1 Source-Structure 1 Reservoir-Structure 1 Line-Structure 6 Independent Arguments-Structure
<b>Geometry Task 4 (GT4)</b>	3 Spiral-Structure 5 Independent Arguments-Structure

When Table 2 is examined with respect to the variety in global argumentation, three kinds of global argumentation structures are observed to have emerged in the solutions for the triangle tasks (GT1 and GT2). On the other hand, in the problems based on the circle, four kinds of global argumentation structures in GT3 and three kinds in GT4 were identified. When the distributions of these structures were examined, it could be observed that the most frequently used global argumentation structure and the one that emerged in all the geometry tasks was the independent arguments-structure. Another structure that emerged in the present study, the line-structure emerged once in GT1 and once in GT2. In order to give a detailed account of these structures, each structure is explained and exemplified below.

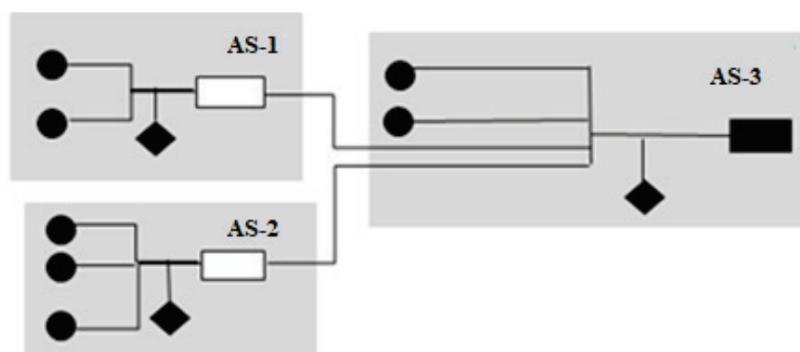
#### *Source-Structure*

Source-structure was observed once in the second and once in the third geometry task. The source-structure observed in GT3 (see Figure 10) is explained and exemplified below. In the diagram in Figure 14, the data provided in the question are shown in black and Gözde's additional drawings are shown in red.



**Figure 14.** The Drawing of GT3 and Gözde's Additional Drawings with Red Colour

The global argumentation structure that emerged in this argumentation can be seen in Figure 15. The argumentation stream is indicated in the diagram with the letters 'AS'.



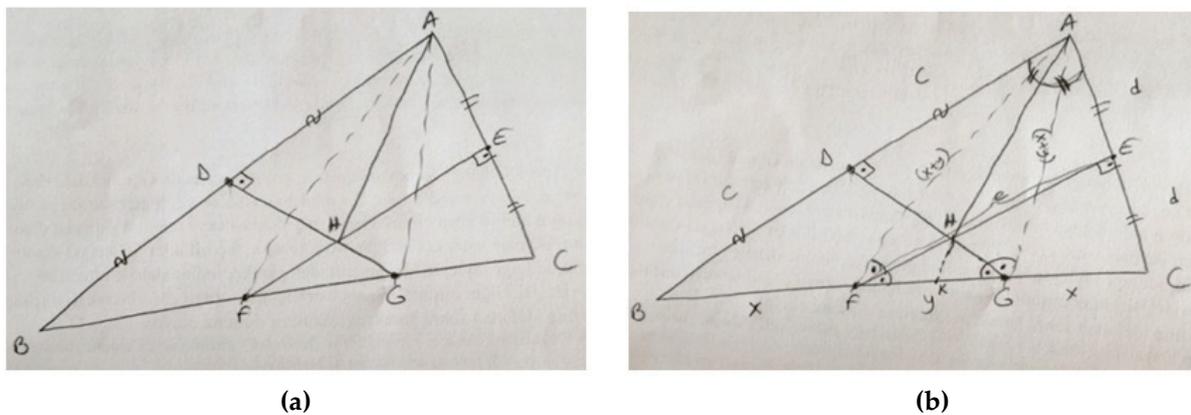
**Figure 15.** Source-Structure Observed in the Solution of Third Geometry Task

By drawing the  $|CH|$ ,  $|HD|$ ,  $|CD|$ ,  $|CE|$  and  $|DE|$  chords in AS-1, Gözde claimed that triangles HCD and CDE were equilateral triangles. She stated that the chords she drew as warrants for this claim were equal to the radius of the circles. Then, in AS-2, she used the angle and arc properties to claim that  $\angle HFG=60^\circ$ . Gözde used her claims in AS-1 and AS-2 as data (claim/data) in AS-3. Furthermore, in AS-3, Gözde, who used the information that ' $\angle HDE=120^\circ$ ,  $\angle HCE=120^\circ$ ' and arc 'HE is  $240^\circ$ ' as additional data, claimed that triangle FGH was an equilateral triangle and calculated its area by using the area formula for the equilateral triangle.

When Figure 15 is examined, it can be observed that arguments generally possess the structure of a funnel, that information derived from different sources is combined to reach a conclusion and they feature the property of a source-structure. Considering that parallel arguments (AS-1 and AS-2) are also present at the beginning of the argumentation, this structure is presented as an appropriate example for source-structure.

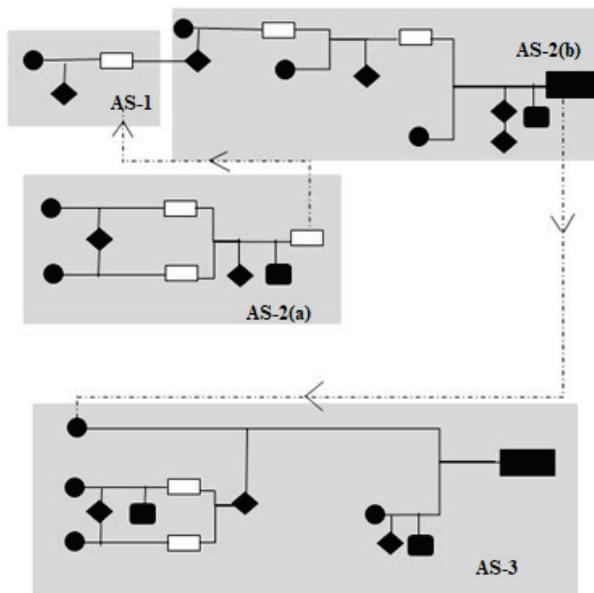
**Reservoir-Structure**

Reservoir-structure was observed once in the first and once in the third geometry task. The reservoir-structure which emerged in GT1, the details of which are portrayed in Figure 8, is explained below. While Güler was solving the problem, she first drew triangle ABC and then placed points F and G on side |BC|, with F on the left and G on the right (see Figure 16(a)). Among the information given in the task, points D and E were stated as mid points. When Güler was drawing these points, she assumed that  $|FE| \perp |AC|$  and  $|GD| \perp |AB|$  and continued to solve the task. Subsequently, she added her drawings of |AF| and |AG| (see Figure 16(a)). Meanwhile, the researcher asked the class to think about whether these two drawings (|DG| and |FE|) could be perpendicular at the same time. This question remained as a question mark in the mind of each participant and Güler continued to work on her geometry task.



**Figure 16.** Güler’s Drawings for GT1 on the Board

The global argumentation structure that emerged in this argumentation is summarized in Figure 17.



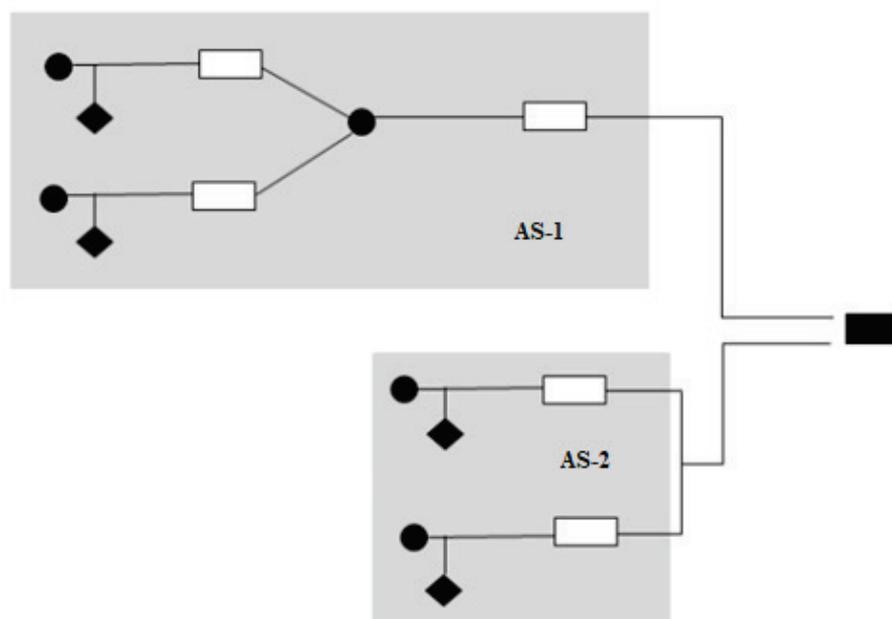
**Figure 17.** Reservoir-Structure Observed in the Solution of First Geometry Task

In AS-1, Güler showed that triangle AFC was an isosceles triangle and then used the properties of isosceles triangles to label  $|AF|$  as ' $x+y$ '. In the same way, Güler used triangle AGB to find that  $|AG|=x+y$  and claimed that triangle AFG was an isosceles triangle. Subsequently, in AS-2(a), Güler again made use of the isosceles triangle properties and claimed that  $|GD|$  was the bisector of  $\angle AGF$  and, likewise,  $|FE|$  was the bisector of  $\angle AFG$ . Based on this data, she arrived at the intermediate conclusion that  $|AH|$  must be the bisector of  $\angle BAC$ . The other participants refuted this argument by claiming that  $|AH|$  was actually the bisector of  $\angle FAG$ . Güler returned to AS-1 to reevaluate her reasoning and continued the argumentation with AS-2(b). Güler intersected segment  $|AH|$  with side  $|BC|$  and labelled the intersection point as K and labelled  $|FK|$  and  $|KG|$  as ' $y/2$ '. By using the information that  $|AK| \perp |BC|$ , she wrote the angle bisector formula for triangle ABC:  $\frac{2c}{x+(\frac{y}{2})} = \frac{2d}{x+(\frac{y}{2})}$ . She argued that only if  $c=d$  would this formula be obtained. Thus, she claimed that only in situations where triangle ABC was an isosceles triangle could  $|AH|$  be an angle bisector. Umut argued that the intersecting three perpendiculars and the three angle bisectors intersected at the same point, and that  $|AH|$  could be an angle bisector only if triangle ABC was an equilateral triangle (AS-2(b)). However, Bahar realized that three of the intersecting perpendiculars did not belong to the same triangle and this argument was refuted as well, so the reasoning proceeded backward to resolve the task. İnci went to the board and went over Güler's solution again, arriving at the conclusion that  $|AH|$  could be an angle bisector when triangle ABC was an isosceles or equilateral triangle (AS-3) and highlighted that the simultaneous presence of the perpendiculars, ' $|FE| \perp |AC|$ ' and ' $|GD| \perp |AB|$ ', mentioned at the very beginning was the only assumption.

In this argumentation, intermediate target conclusions were observed and owing to those refuting the reasoning, she had to proceed backward twice to question the claims. For this reason, it was concluded that this argument was compatible with the reservoir-structure.

#### *Spiral-Structure*

Spiral-structure was observed twice in the second geometry task, and three times in the fourth geometry task. The spiral-structure observed in GT4 (see Figure 11) is explained as an example below. Özer and Beren, who discussed the first question of the task amongst themselves, used two different methods to show the equality  $|EH|=|CE|$ , and the structure of the argumentation emerged as displayed in Figure 18.



**Figure 18.** Spiral-Structure Observed in the Solution of Fourth Geometry Task

In the first argument in AS-1, Beren initially made the following labellings:  $|CE|=|ED|=a$ ,  $|FE|=2b$ ,  $|EG|=2c$  (see Figure 19). Then she wrote ' $|CE|.|ED|=a.a=2b.2c=4bc$ ' by indicating *the intersecting chords theorem* as a warrant to arrive at the conclusion that  $|CE|$  is equal to  $\sqrt{4bc}$ . Meanwhile, Özer did the red-colored drawings below. Özer arrived at the conclusion that  $|EH|$  is also equal to  $\sqrt{4bc}$  by applying *the Pythagorean theorem* in the right triangle HEO. As the claims in both of these arguments are data (claim/data) for the target solution ' $|EH|=|CE|$ ', they are depicted as hollow rectangles.

In the first argument in AS-2, Özer combined  $|HF|$  and  $|HG|$  and realized that angle FHG stands on the diameter of the semi-circle. Expressing this as a warrant, he claimed that angle FHG was  $90^\circ$ . In the second argument, Beren realized the perpendicular from point H to the hypotenuse in the right triangle FHG and stated that Euclid's formula could be used in this triangle. Then she wrote the formula  $t^2=2b.2c$  and arrived at the conclusion,  $|EH|=\sqrt{4bc}$ . Thus, she arrived at the conclusion ' $|EH|=|CE|$ ' by using a different method of solution.

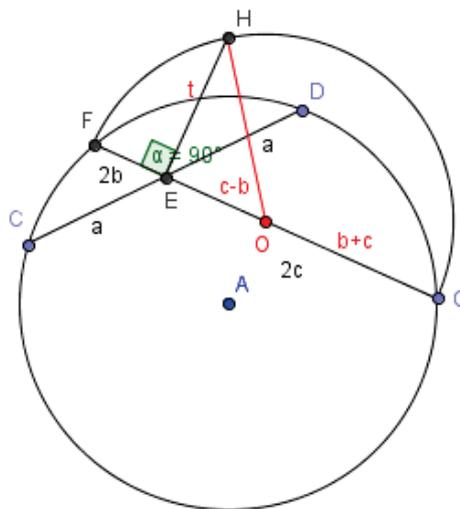


Figure 19. Drawings of Özer and Beren on the Shape of GT4

In the present argumentation, two parallel argumentation streams, AS-1 and AS-2, arrived at the same target conclusion ( $|EH|=|CE|$ ) by means of two different methods of solution. That is, parallel arguments are located at the end of the argumentation process. For this reason, it was concluded that this structure was compatible with the spiral-structure.

#### Line-Structure

Line-structure was observed once in the first and once in the third geometry task. The example that can be given for this structure emerged in the solution to the second question of GT3. The details for GT3 were explained in the section on findings related to source-structure. In the first question of this task, the length of side  $|FG|$  was given as 6 units and the area of triangle FGH was asked. On the other hand, in the second question, the maximum and minimum values for the area of triangle FGH when the radius of each circle was accepted as ' $r$ ' were asked. Both Gözde and Umut found that triangle FGH was an equilateral triangle in their solution of the first question. In the solution for the second question, the line-structure in Figure 20 emerged.

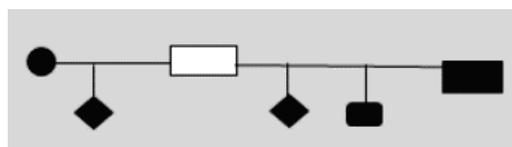


Figure 20. Line-Structure Observed in the Solution of Third Geometry Task

The dialogue exchanged between Umut and Gözde is as follows:

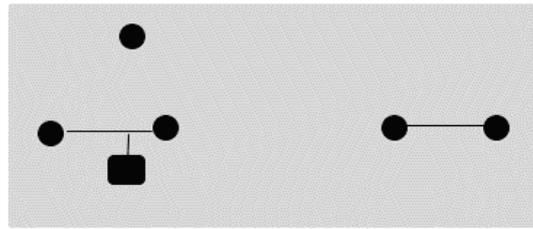
- Umut* : That's right. FGH is always an equilateral triangle (Claim/data). We used 6 units given in the question in the last stage. First, we found it was an equilateral triangle (Warrant). What is  $\sin 60^\circ$ ?
- Gözde* :  $\sqrt{3}/2$ . When we obtain a triangle with maximum area, I think one side is as long as  $r$  (Claim).
- Umut* : What was the sine area formula? Was it  $(1/2) \cdot x^2 \cdot \sin 60^\circ$ ? Its area will change according to this  $x$  anyway. I mean what is its derivative? The maximum that  $x$  can be is  $r$ . That's it,  $r \cdot \sqrt{3}/2$ .
- Gözde* :  $2r \cdot \sqrt{3}/4$ . Why did you take its derivative?
- Umut* : Hmm... Yes, you're right.
- Gözde* : You found the enlargement ratio. If you take the derivative ...
- Umut* : Oh, not derivative. What was the maximum, minimum value? Was it the second derivative?
- Gözde* : Then, it is  $2\sqrt{3}/4$ .
- Umut* : Isn't the area of the triangle  $x^2 \cdot \sqrt{3}/4$ ? The first derivative of this is  $2x \cdot \sqrt{3}/4$ . The second derivative is  $2\sqrt{3}/4$ . So it's  $\sqrt{3}/2$  isn't it? I mean, isn't it this?
- Gözde* : Weren't we to place the value in its place? I didn't understand that.
- Umut* : Yes. We found a value. We couldn't find a minimum value though. The minimum value would be zero anyway. And the maximum value... Am I mistaken? Still, I think this problem is solved using the derivative.

Umut believed that chord  $|EF|$  should be dragged so that the triangle could reach the maximum area it could possess. Subsequently, he claimed that triangle FGH was an equilateral triangle in all conditions (claim/data). As a warrant for this claim, Umut stated that, while arriving at the solution that triangle FGH was an equilateral triangle in the first task, they had not made use of the 6 units given in the question. By using this information as data, they passed onto the next argument. When they obtained the triangle with a maximum area, they argued that one side of the triangle could be as long as ' $r$ ' and they tried to devise a formula by taking the derivative of the sine area formula ' $(1/2) \cdot x^2 \cdot \sin 60^\circ$ '. After finding the derivative to equal ' $2x \cdot \sqrt{3}/4$ ', they wrote  $r$  in place of  $x$  as one side of the triangle was equal to ' $r$ ' and calculated the area. In this task, as a target conclusion, both Gözde and Umut claimed that the maximum area of triangle FGH was  $r\sqrt{3}/2$ . As a warrant to this claim, they used the information that triangle FGH was always an equilateral triangle and additionally, took the derivative of the sine area formula. However, it should be noted that Gözde refuted the claim by arguing that to find the maximum area, the second derivative rather than the first derivative of the formula needs to be taken. The argumentation ended at this point as it deviated into another direction.

When the structure of this argumentation is examined, it is observed that the arguments are sequenced in a linear fashion, that they are connected with the claim/data components. Furthermore, this structure is distinguished from the other structures based on the lack of parallel arguments and the lack of back and forth reasoning. As this argumentation schematically proceeds on a line in the argument diagram, it was decided that this argument was compatible with the line-structure.

#### ***Independent Arguments-Structure***

The independent arguments-structure was observed 8 times in GT1, twice in GT2, 6 times in GT3 and five times in GT4. The sample details to be given for this structure were observed in the solution for GT1, which is explained in Figure 2. In this task, the participants were required to find in which types of triangles  $|AH|$  would be an angle bisector. Arguments presented at the beginning of the argumentation were presented independent of each other, as displayed in Figure 21.



**Figure 21.** Independent Arguments-Structure Observed in the Solution of First Geometry Task

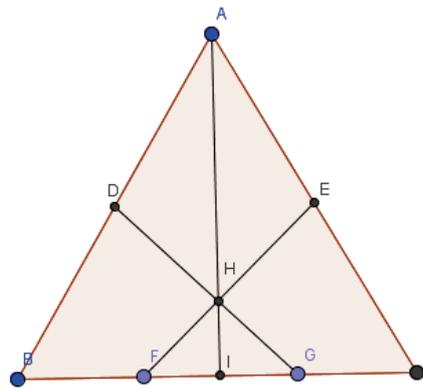
The following dialogue was exchanged between Bahar and Güler on the three independent arguments in Figure 21:

*Bahar* : Points  $F$  and  $G$  can be placed in two separate ways. In my opinion, they can change places (Claim 1).

*Güler* : Let me draw segment  $|AH|$ . If we label this as  $a$ , and  $x$  as  $x\dots$  Oh, yes, right. Then this is an isosceles triangle. (Claim 2)

*Bahar* : Why did you think that? This is not an isosceles triangle. (brief silence)

*Güler* : I think we should draw an equilateral triangle and analyse it on that. Let's extend  $|AH|$ . Let this be  $T$ . Then,  $|AI|$  would be the angle bisector of both  $BAC$  and  $FHG$  triangles, wouldn't it? (Claim 3)



**Figure 22.** Drawings of Güler and Bahar for GT1

The first argument in Figure 21 is comprised merely of the claim component; Bahar claims that the places of points  $F$  and  $G$  can change. Then Güler claims that when triangle  $ABC$  becomes an isosceles triangle,  $|AH|$  can be the angle bisector. Bahar ends this claim by saying that triangle  $ABC$  is not an isosceles triangle without presenting any warrant. After thinking for a while, Bahar and Güler decided to draw an equilateral triangle and work on that (see Figure 22). They labelled the point at which  $|AI|$  intersected  $|BC|$  with the letter  $T$  and claimed that  $|AI|$  was the angle bisector of both  $BAC$  and  $FHG$  (see Figure 22), but they did not provide warrant for these claims.

As can be observed in this discussion, participants proposed arguments that were not related to each other. Actually, these arguments were produced by the participants while they were thinking aloud. As they were schematically disconnected arguments, the decision that this structure was compatible with the independent argument-structure was made.

## Discussion, Conclusion and Suggestions

The present study aimed to identify the global argumentation structures utilized by prospective middle school mathematics teachers while solving geometry problems. There are no explanations reported by studies in the related literature of one global argumentation structure being superior to another. It has only been stated that some structures are figuratively more complex when compared to others. Complex global argumentation structures emerge as a result of employing higher order thinking skills as well as with comprehensive examination and association of concepts. Three of the global argumentation structures that emerged in the present study were, as mentioned in their studies by Knipping (2008) and Reid and Knipping (2010), *source-structure*, *reservoir-structure* and *spiral-structure*, and two of them were *line-structure* and *independent arguments-structure*, which are concepts contributed to the literature by means of the present study. The findings of the study revealed that the participants made low use of complex global argumentation structures such as source-structure, reservoir-structure and spiral-structure. It was observed that the participants mostly used the independent arguments-structure, and among the structures present in the literature, they mostly used the spiral-structure. Following is an account of the foresights regarding the use of these structures.

*Mathematical reasoning* has been defined in the literature as the skill to establish a relationship among concepts and to make reasoned inferences based on these relationships (Conner et al., 2014). Based on this definition, it can be deduced that mathematical reasoning is based on the competence to establish mathematical relationships. However, complex relationships cannot be established among the relatively simple structures as line-structure and independent arguments-structure when compared to the argumentation structures present in the literature. In the independent arguments-structure, where higher order thinking skills are not employed, arguments are expressed individually. With the examination of the contexts in which these argument structures were used, it was noticed that the participants constructed arguments that did not have any relation to one another as such while they were thinking aloud. Furthermore, while reasoning, the participants used these structures more at the beginning of problem-solving task. The fact that prospective teachers did not make associations among concepts, even if it were at the beginning of the problem-solving process, reveals their weaknesses in mathematical reasoning. Based on this finding, it can be claimed that the mathematical reasoning skills of prospective middle school mathematics teachers are weak. On the other hand, it is evident that the participants used the spiral-structure more than the other global argumentation structures (source-structure and reservoir-structure) present in the literature. Considering the properties of spiral-structure, the presence of different methods of solution, which are parallel to each other (parallel argumentation streams) and which reach the target result can be observed. The most important reason why these structures are observed may be related to the way the first researcher directed the argumentation process. In a study on proof by Knipping (2008), it was advocated that the teacher's instructions are effective in shaping the global argumentation structure. Likewise, the researcher of the present study continuously encouraged the participants to show different methods of solution and continuously questioned their responses throughout the argumentation process, thus creating the opportunity for the establishment of parallel argumentation streams. When these parallel streams are generally displayed on a diagram, a spiral-structure is observed to be formed. If the researcher had not guided students in this way, the student may have responded to the questions by providing one method of solution and the shape of the argumentation structures would have been in the form of line-structure. Thus, it is revealed that the key element in the formation of argumentation structures is the person who guides the argumentation process.

Based on these findings, conclusions can be drawn regarding the training of prospective teachers in teacher education institutions. A teacher with developed argumentation skills questions students to constantly provide warrants during the argumentation process, encourages them to show different methods of solution, avoids making judgemental comments, encourages student interactions and follows student discussions carefully to guide them by asking the most appropriate questions. An effective realization of these mentioned practices can be achieved by providing prospective teachers

with effective training in this area. Moreover, by providing appropriate guidance, well-trained prospective teachers can establish environments in which students can generate different methods of solution, present relationships among concepts, and use complex global argumentation structures. Hence, during their undergraduate study, prospective teachers should be provided with environments in which they can develop their argumentation skills. In this regard, the courses offered in the teacher education programs in Turkey need to be restructured to aim for the development of argumentation skills of prospective teachers. Implemented more commonly in the science teacher education program, the argumentation process needs to be made more commonly used in the maths teacher education program as well. This is highly important as, in this way, the argumentation method can also be applied in schools in the future. Practices such as discussing a topic or problem, supporting arguments with warrants, evaluating the arguments of others, and rebutting when necessary will not only help students see the relationships between concepts and engage in conceptual learning (Inagaki et al., 1998) but also enable them to develop their higher order thinking skills (Kosko et al., 2014). Based on this reason, it is recommended that argumentation practices be incorporated into both must and elective courses in teacher education institutions. In this way, mathematics teachers can gain experience in argumentation and can implement it in their lessons when they begin to pursue their profession.

Mathematical reasoning, which is primarily based on using mathematical associations, is quite important with respect to argumentation; it is a skill that individuals learn and develop over time. Based on the findings of the research, the reasons underlying the simplicity of the global argumentation structures can be attributed to the educational background of prospective middle school mathematics teachers, the national exams and the education programs implemented in Turkey. National exams in Turkey (for example the University Entrance Exam, UEE) is a multiple-choice exam prepared at a standard whereby students can be successful by memorizing abstract information and rules and by practising effective multiple-choice test solving techniques (Güler, 2013). Hence, the education offered at schools aims for these students to be successful in these kinds of exams. Consistent with this aim, students are trained at schools to solve more and more questions in shorter periods of time. For this reason, throughout their educational life, students cannot gain sufficient experience in practices such as providing warrant, thinking reflectively, providing proof, and discussing relationships among concepts, which develop mathematical reasoning and support learning (National Council of Teachers of Mathematics, 2000). Therefore, students cannot think critically when they come across a question that they are unfamiliar with but need to comment on. As the prospective middle school mathematics teachers are products of the Turkish education system too, it is believed that they too did not gain experience in activities catering to develop their mathematical reasoning and, hence, have the tendency to use simple global argumentation structures. Accordingly, it can be claimed that mathematics education in Turkey is quite short of developing students' mathematical reasoning skills and, thus, needs to be improved. In this regard, the step that could be taken is to reconstruct mathematics education programs. Activities implemented in schools can be rearranged so that they are based on argumentation and questioning. In addition, national exams could be prepared by including not only multiple-choice questions but also open-ended or short response questions to support the education programs in schools. In this way, contributions can be made to developing students' higher order thinking skills and critical thinking skills (Weiss, 2003). The open-ended and multiple-choice questions of such exams as TIMSS (Trends in International Mathematics and Science Study), which compares mathematics achievement on an international scale, and PISA (MEB, 2015a, 2015b) can be examined to see what kind of questions can be prepared in national exams because such international exams are based on questions that are associated with daily life, which measure higher order thinking skills; it is not possible to be successful in these exams by merely memorizing information.

As for the limitations of the present study, it can be observed that the study is limited to four geometry questions solved in the topics of triangles and circles. In order to make inferences on argumentation practices in geometry and examine further the various methods of argumentation, more argumentation studies on such geometry topics as quadrilaterals, polygons, geometric objects and angles need to be conducted. To this end, it is recommended that activities comprehensively addressing

other geometry topics be prepared and implemented in the area of argumentation process. For example, activities on polygons and their rules can be prepared and implemented so as to support prospective teachers' argumentation process and enable them to question these rules more profoundly and establish relationships among concepts. In this way, by analyzing argumentation processes of prospective teachers in the area of polygons, what global argumentation structures they use can be examined. Another limitation of the present study is the selection of middle school mathematics teacher candidates enrolled in a state university in Ankara to form the sample group of the study. The argumentation process can vary across universities. Hence, in order to be informed about the conditions in different universities, similar argumentation studies should be conducted in other universities as well. It is believed that this will enable us to find out what kinds of benefits or negative consequences the argumentation process may have in different universities and to make general judgements about what kinds of implementations will yield productive outcomes.

As argumentation is a method that is based on individuals' presentation of warrants for their own claims, convincing their opponents, and refuting others' opinions, it can produce different results in different cultures (Reid & Knipping, 2010). To illustrate, in such countries as Japan, where people communicate harmoniously and in a well-mannered way, and where opposing someone is seen as a threat to harmony, it would be quite difficult to successfully implement the practice of argumentation (Sekiguchi, 2000). For this reason, it is believed that conducting argumentation studies on an international scale will be beneficial to reveal certain global argumentation structures peculiar to different cultures and to gain insight into whether or not the argumentation method can be practised in different countries.

As a final note, Knipping (2008), who proposed the global argumentation analysis, maintained that in this analysis, the general or global diagrams produced by interrelated arguments are evaluated, while individual contributions are not analyzed. In order to examine the components of arguments (data, claim, warrant etc.) individually and to draw conclusions, it is believed that examining participants' individual arguments will be of benefit. The analyses of participants' individual contributions to the argumenation process can be the subject of future research studies.

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