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Components of Mathematical Problem Solving Competence and Mediation Effects of Instructional Strategies for Mathematical Modeling

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Abstract

This study is aimed at investigating the relationships between the components of problem solving skills (four procedural components and modeling competence) and the impact of instructional strategies (collaborative learning and problem posing) on students' mathematical modeling competence. A survey on students' mathematical problem solving competence, that was comprised of 40 items, was administered to 1,224 students in Korea and their responses were analyzed quantitatively (exploratory factor analysis, confirmatory factor analysis, and path analysis). The results indicated that students' competencies regarding procedural components of mathematical problem solving positively affected their mathematical modeling competence. Additionally, instructional strategies utilizing collaborative learning and problem posing mediated and synergized the effect in terms of procedural components of mathematical problem solving and mathematical modeling competence. The study also discusses its contributions to and implications for mathematics educators and teachers.

Keywords

Mathematical problem solving
Mathematical modeling
Instructional strategy
Structural equation modelling

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Introduction

In the 21st century, developing students' mathematical competencies is becoming a critical topic, especially with the advent of the 4th industrial innovation. As the 4th industrial innovation emerged, mathematical competencies, which may be useful in workplaces that deal with complicated and non-routine problems, have been emphasized more. In other words, for current and future generations, it would be more critical to apply mathematical content than to memorize it. In the Programme for International Students Assessment (PISA) report, the Organization for Economic Cooperation and Development (OECD, 2014) indicated that "today's workplaces demand people who can solve non-routine problems" (p. 26), and also reported that almost 10 percent of workers confronted complex, non-routine problems every day in their workplaces (OECD, 2014). Computers or artificial intelligence will take over most repetitive tasks for the generation of the 4th industrial innovation, resulting in humans playing the role of a leader requiring pivotal, mathematical competencies such as mathematical problem solving and modeling.

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Since mathematical problem solving skills and modeling competencies are considered important, mathematics educators are advised to prepare to cultivate students' mathematical competence. National Council of Teachers of Mathematics (NCTM) stated that "problem solving must be the focus of school mathematics" (NCTM, 1980, p. 1) and emphasized that mathematical problem solving is one of the primary goals in mathematics education. OECD (2006, 2009) also indicated a list of mathematical competencies in the PISA reports that an individual needs to possess, including mathematical problem solving and modeling competencies. To satisfy these goals in mathematics education, the emphasis has shifted from encouraging students to memorize mathematical content to preparing them with problem solving skills for complex, non-routine, and real-world problems. Moreover, much effort has been invested into research to find instructional strategies that improve students' mathematical competencies, such as problem solving and modeling (Albaladejo, Garcia, & Codina, 2015; Blum & Ferri, 2016; Callejo & Vila, 2009; Lester, 2013; Schukajlow, Kolter, & Blum, 2015).

To further advance the strategic, instructional approach for developing students' mathematical competencies, it helps to provide mathematics teachers and educators with more information regarding the relationships among the components of mathematical competencies and the effects of instructional strategies on mathematical competencies (Han, Cetin, & Matteson, 2016; Park, Jang, Chen, & Jung, 2011). Previous studies have documented the effects of instructional strategies on academic achievement in mathematics instead of mathematical competence (Capraro, Capraro, Yetkiner, Rangel-Chavez, & Lewis, 2010; Kajamies, Vauras, & Kinnunen, 2010; Wiliam, Lee, Harrison, & Black, 2004). In addition, most studies exploring the impact of the instructional approach on students' mathematical competence used qualitative analysis or simple statistics (Blum & Ferri, 2016; Serin, 2011; Tiwari, Lai, So, & Yuenm 2006). However, there have been limited studies using the advanced quantitative analysis approach to investigate the extent to which these instructional strategies affect students' mathematical problem solving and modeling competencies, and how each mathematical competence influences other mathematical competences. Therefore, the current study aims to investigate the procedural components of problem solving, instructional strategies (i.e., collaborative learning and problem posing), and mathematical modeling, and the relationships among them. The findings from the study will provide mathematics teachers and educators with information on mathematical competence and instructional strategies for achieving it.

Literature Review

Mathematical Problem Solving

Mathematics is not simply the process of acquiring facts, processes, formulas, and principles, but a process of creative thinking and logical reasoning, as well as exploring problems that require communication (Baroody & Coslick, 1998). For students, mathematical problem solving is a high-level learning process that can change a problematic situation and creatively address it through the application of rules and procedures (Sivunen & Pehkonen, 2009). It is a process of self-examination, where students solve the problems on their own, and this process requires sufficient mathematical thinking experience (Pólya, 1957).

Many researchers have emphasized the importance of problem solving as a means of developing the logical thinking aspect of mathematics (Kilpatrick, 2009; Schoenfeld, 1985). Most mathematical processes involve problem solving and it has been an important goal of mathematics education, since the NCTM emphasized the importance of improving mathematical problem solving skills in 1980. Mathematical problem solving skills have been emphasized not only in terms of the practical and instrumental application of mathematics in everyday life and to other subjects, but also to cultivate mathematical reasoning and acquire methods of mathematical investigation.

The discussion on mathematical problem solving was borne from the discussion on problem solving. Researchers have elucidated the difference between the meaning of problem solving and *mathematical* problem solving (Kilpatrick, 2009; Pólya, 1957). According to Pólya (1980), problem solving is "to find a way where no way is known, off-hand, to find a way out of a difficulty, to find a way around an obstacle, to attain a desired end, that is not immediately attainable, by appropriate means" (p. 1), and described the process of mathematical problem solving in terms of four stages: understanding the problem, devising a plan, carrying out the plan, and looking back. Pólya (1980) mentions that there are purely mathematical problems and practical problems involving mathematical problems. Whether the problem is mathematical or practical, the primary motives and procedures for finding the solution must be the same (Pólya, 1957). Kilpatrick (2009) tried to differentiate *mathematical* problem solving from problem solving by stating that mathematical concepts and principles should be used in seeking the answer. Later, the concept of mathematical problem solving was broadened by Lesh and Zawojeswski (2007). They added models-and-modeling perspectives to problem solving, and emphasized that mathematical problem solving is a complex activity that goes beyond the realm of school mathematics and involves iterative cycles of expressing, testing, and revising mathematical interpretations.

Just as the definition of problem solving is diverse, previous studies have also described the process of problem solving in various ways. Researchers (Anderson, Lee, & Fincham, 2014; Schoenfeld, 1985) claimed that the actual problem solving process is often creatively and complicatedly composed and functions in accordance with the schema that is crucial and available to the student, instead of linearly as in the model. Subsequent studies made modifications to Pólya's problem solving model based on this perspective. One of them is the problem solving model by Schoenfeld (1985). Schoenfeld further divided Pólya's model and added the investigating process to formulate a plan for addressing difficult problems. This process involves making modifications and supplementations by selecting and applying appropriate strategies based on previous experience. In addition, OECD (2014) indicated the four phases–exploring and understanding, representing and formulating, planning and executing, monitoring and reflecting–as the main cognitive processes in solving a problem in the 21st century. Whereas, Anderson et al. (2014) discovered five critical, cognitive events (define, encode, compute, transform, and respond) when solving mathematics problems by detecting brain signatures and mousing behaviors. Although these studies suggest various problem solving processes, they commonly involve Pólya (1957)'s four steps. Therefore, this study, based on the four-step problem solving theory of Pólya (1957), defined the mathematical problem solving process as understanding the problem, devising a plan, carrying out the plan, and looking back.

Mathematical Problem Solving Competence

As the concept of mathematical problems changed, the goals of mathematics education have also continuously changed. The current mathematics education aims to prepare students for mathematical problem solving skill or competence, whereas the goal in traditional mathematics classrooms was to make students master the facts and procedures of mathematical techniques (Cho & Kim, 2013; Larmer, Ross, & Mergendoller, 2009). To achieve the goal pursued in the past, problems were mostly used as routine exercises (Schoenfeld, 1992). In other words, mathematics problems were routine exercises organized to provide practice on a particular mathematical technique that had just been demonstrated to the students. Similarly, mathematical problem solving was also regarded as the technique of "[being] taught as subject matter with practice problems assigned so that the techniques can be mastered" (Schoenfeld, 1992, p. 338). In the second half of the 20th century, as mathematics problem solving received more attention from mathematics educators, mathematical problems included not only routine exercise, but also practical, non-routine, and real-world problems (NCTM, 2000). As a result, mathematics education has come to pursue the goal of equipping students with problem solving skills rather than mastering the facts and procedures of mathematical techniques.

Problem solving has sometimes been regarded as *skill*, and sometimes as *competence*. According to Stanic and Kilpatrick (1988), problem solving already includes the concept of skill within it and thus, students could be trained using school mathematics curriculum. Therefore, the term *mathematical problem solving skill* is used in the field of current mathematics education to emphasize the aspect that mathematical problem solving is educatable skill. The NCTM (2000) defined the mathematical problem solving skill as the ability to understand the already known mathematical concepts, principles, and rules, and perform various and comprehensive thinking process, such as mathematical algorithm, discoverology, and strategy, based on knowledge or skills to address a given problematic situation.

On the other hand, the concept of *competence* goes beyond the simple meanings of knowledge or skills. It involves satisfying more complex requirements by implementing the skills as well as mental resources, such as an attitude, in a given specific situation. For instance, competence is the ability to facilitate effective communication, based on verbal fluency, information and communication technique applicability, or the mental attitude toward others (Rychen & Salganik, 2003). Based on the meaning of competence, problem solving competence has been regarded as "the ability to figure out a solution for a solution method for reaching one's goal if no such method is obvious" (Fischer et al., 2015, p. 172). In addition, OECD (2014) used the term *problem solving competence*, and defined it as "…an individual's capacity to engage in cognitive processing to understand and resolve problem situation where a method of solution is not immediately obvious. It includes the willingness to engage with such situations in order to achieve one's potential as a constructive and reflective citizen" (p. 30). OECD (2014) extended others' concept of problem solving competence (Duncker, 1945; Wirth & Klieme, 2003) by including the aspect of willingness to solve a problem. Bruder (2000) also extended the concept of mathematical problem solving competence by moving the concept of problem solving competence to a mathematical context. She defined it as the ability to simplify the problematic situation, contemplate proper stages of problem solving, and consider various aspects simultaneously to address the problem while restructuring the mathematical facts.

The mathematical problem solving competence has recently been a goal of mathematics education, in addition to mathematical problem solving skills. Therefore, in the current study, we have used the term "mathematical problem solving competence," to comprehensively represent knowledge, skill, and attitude for mathematical problem solving. The mathematical problem solving mentioned in this study is the application of strategies and functions that could be helpful in addressing the problems faced while learning mathematics or in daily life. The problem solving competence in terms of mathematics and its context include the ability to utilize previously acquired mathematical knowledge as well as technique, attitude, and strategies for effective execution of tasks, and inspect the derived knowledge during the problem solving process through reflective consideration.

Mathematical Modeling

The concept of mathematical modeling is multifaceted. Among the various aspects of mathematical modeling, the focus has been on mathematical modeling being a thought process connecting the real world to mathematics (Blum & Ferri, 2016; OECD, 2009). Common Core State Standards Initiative (2014) described mathematical modeling as "using mathematics or statistics to describe (i.e., model) a real-world situation and deduce additional information about the situation by mathematical or statistical computation and analysis" (p. 5). That is, in a mathematics lesson dealing with mathematical modeling, students are required to translate real world situation into mathematical form using mathematical terms, representations, and models. In the same vein, Pollak (2003) documented that mathematical modeling is distinguished from other mathematical activities in terms of "explicit attention at the beginning of the process of getting from the problem outside of mathematics to its mathematical formulation, and an explicit reconciliation between the mathematics and the real-world situation at the end" (p. 649).

Moreover, the dynamic process of mathematical modeling has been emphasized (Bliss, Fowler, & Galluzo, 2014; Dossey, McCrone, Giordano, & Weir, 2002). There has been no agreed upon procedure for mathematical modeling, however, literature commonly indicates that the process of mathematical modeling is not linear but cyclical and that it includes multi phases (Common Core State Standards Initiative, 2014; NCTM, 1989). For example, OECD (2006, 2009) listed the phases of mathematical modeling as "structuring the field situation to be modelled; translating reality into mathematical structures; interpreting mathematical models in terms of reality; working with a mathematical model; validating the model; reflecting, analyzing and offering a critique of a model and its results; communicating about the model and its results; and monitoring and controlling the modeling process" (p. 97). That is, in applying mathematical modeling, students may begin in a particular phase but resume to any other phases based on their need. The critical point of the mathematical modeling cycle is that mathematical modeling includes the phases that convert the actual situation into a mathematical problem, seek a solution to the problem, and then interpret the solution in accordance to the initial actual problem again (Cirillo, Pelesko, Felton-Koestler, & Rubel, 2016; NCTM, 1989).

Mathematical modeling has also been illuminated as a whole process including *problem finding* as well as *problem solving* (Pollak, 2012). Mathematical modeling possesses not only the problem solving phase, but also the phase of finding a problem from a real world situation. Identifying a problem from the real world and converting it into mathematical form is not a task that has often been implemented in mathematical classrooms where mathematics problems are usually given to students. For this reason, mathematical modeling tasks contain high cognitive barriers and demand students' diverse thinking abilities such as making assumptions and decisions, optimizing a situation, interpreting results, and modifying a solution (Blum & Ferri, 2016). This explains how mathematical modeling is different from the other mathematical tasks and why mathematical modeling needs to be incorporated into mathematics lessons.

To sum up the features of the mathematical modeling described so far, mathematical modeling may be compared with mathematical problem solving in the following manner. Mathematical modeling starts from and comes back to real-world situations whereas mathematical problem solving might comprise real-world situations or pure mathematical problems (Kim, 2012; Pollak, 2012). Although mathematical problem solving and modeling commonly refer to the real world, the real world in mathematical problem solving is more likely to be idealized rather than unedited. In addition, mathematical problem solving may include pure and applied mathematical problems, whereas mathematical modeling is basically used for the application of mathematics using non-mathematical situations (Blum & Niss, 1991). Moreover, mathematical problem solving tends to be given to students after they learn mathematical concepts and formulation, whereas mathematical modeling is utilized in teaching students mathematics (Lesh & Zawojewski, 2007). For this reason, mathematical modeling involves tasks that demand a high level of cognition whereas mathematical problem solving covers tasks that demand diverse levels of cognition.

Based on this comparison between mathematical problem solving and modeling, in the current study, mathematical modeling competency was regarded as a subcomponent of mathematical problem solving competency. Therefore, in the following sections, the concept of mathematical problem solving would include mathematical modeling.

Instructional Strategies

Several pedagogical approaches have been shown to help students improve their problem solving abilities. To understand what instructional strategies are useful for students' mathematical problem solving skills, it is important to see how students learn mathematics. Since the 20th century, the instructional approach for mathematical problem solving has been based on constructivism. As constructive researchers, Pólya (1957) and Pólya and Szegö (1978) conceptualized mathematics as an *activity* and argued that mathematics is similar to science, which depends on guessing, insight, and discovery. Fundamentally, in a constructivist learning environment, several teaching-learning strategies that encourage the process of students constructing mathematical knowledge are critical when adapting these scientific phases into the mathematical problem solving process (Capraro, Capraro, & Morgan, 2013). For example, students' collaboration with peers and teachers is the basic tool that should be employed in constructivist classrooms (Kaldi, Filippatou, & Govaris, 2011; Lou, Liu, Shih, Chuang, & Tseng, 2011; Van Rooij, 2009). Similarly, mathematical problem posing would help students construct knowledge on the problem and problem solving process (Crespo & Sinclair, 2008; NCTM, 1989; Yuan & Sriraman, 2011). Since the current study regards these two instructional strategies as critical in improving students' mathematical problem solving skills, it will now review the literature on collaborative learning environment and problem posing.

Students' group collaboration was examined as a critical instructional approach for students' problem solving skills (Lester, 2013). Many researchers have examined the effects of collaborative problem solving lessons (Kaldi et al., 2011; Lou et al., 2011; Tausczik, Kittur, & Kraut, 2014; Van Rooij, 2009). Facilitating individuals to collaborate in solving problems enables students to share information, build rich outcomes, and accomplish complex tasks (Cranshaw, & Kittur, 2011). Another study (DiDonato, 2013) observed that collaborative, interdisciplinary, authentic tasks positively affected students' self-regulated learning processes, which in turn improved students' problem solving competence. However, the study does not investigate how and why students' collaboration in groups contributes to students' problem solving competence. In other words, there have been few studies exploring the process structuring the impact of collaborative instructional strategies on students' problem solving competence. Therefore, the current study theorizes the structure between a collection of problem solving procedures and modeling competencies and examines the mediation effect of collaborative teaching and learning strategies.

Problem posing has been highlighted as an important method of classroom instruction on problem solving (Crespo & Sinclair, 2008; NCTM, 1989; Yuan & Sriraman, 2011). Problem posing in mathematics classrooms usually refers to either generating new problems or reformulating given problems (Silver, 1994). Specifically, reformulating given problems occurs in the process of solving problems by changing the context or switching some of the information in order to get the solution (Silver, 1994). That is, problem reformulation might help students understand the problem and find adequate strategies for solving it. However, the phase of problem solving process that problem posing affects positively has not been determined. To improve students' problem solving, it will be necessary to discover the impact of problem posing on students' competence for each phase of problem solving.

Unfortunately, there has been limited research on pedagogical knowledge that teachers must possess when teaching mathematical problem solving (Lester, 2013). The findings from the current study could contribute to understanding the instructional approach that would be useful for teachers who need to train students in mathematical problem solving skills. The current study aims to examine how the students' problem solving skills affect their mathematical modeling competence.

Measurement Instrument for Mathematical Problem Solving

As students' mathematical problem solving gained attention, mathematics educators are trying to develop an instrument to measure this skill by developing instructional strategies for helping students solve mathematical problems. Kloosterman and Stage (1992) constructed and validated a Likert-type instrument to investigate students' beliefs on mathematical problem solving. Shute, Wang, Greiff, Zhao, and Moore (2016) employed an engaging video game to measure students' problem solving skills. However, the development of measurement instruments for mathematical problem solving competence has been limited.

In many studies evaluating students' mathematical problem solving competence, the process validating the measurement instrument was omitted. For example, in the study by Bicer, Capraro, and Capraro (2013), a measurement instrument was employed to test the effectiveness of the instructional approach in improving students' mathematical problem solving; however, there was no verification on the test items in terms of validity. In some cases, general items for evaluating academic achievement in mathematics were used where students' mathematical problem solving competence needed to be evaluated. In the study by Elliott, Oty, McArthur, and Clark (2001), an interdisciplinary algebra/science course was examined in terms of the impact on students' problem solving skills. They also had two student groups (interdisciplinary course and college algebra course) and compared their pre- and post-scores on the common tests. In this study, midterm and final tests items were used to evaluate students' mathematical problem solving competence. Although there is a high correlation between problem solving competence and mathematics performance (correlation coefficient=0.81, OECD, 2014), mathematical problem solving competence needs to be differentiated from the mathematical performance measured by midterm or final tests on mathematics. Whereas test items measuring mathematical performances are relatively more likely to emphasize students' mathematical knowledge and skills, mathematical problem solving competence is a concept that goes beyond simple mathematical knowledge and skills. Therefore, the measurement tool for mathematical problem solving competence must be different from tests for mathematics knowledge and skills.

Moreover, most measurement instruments employed in previous studies on mathematical problem solving did not effectively investigate students' mathematical problem solving skills for each phase. For example, even students who represent the same level of mathematical problem solving skills with a test including mathematics practice problems might have different levels of competency for each phase of the mathematical problem solving process (Wilson, Fernandez, & Hadaway, 1993). Therefore, it would be necessary to develop a measurement tool to measure separate phases of mathematical problem solving skills. To satisfy this need, the current study developed a tool that measures students' problem solving skills at each phase.

Research Design/Hypotheses

The current study explored the following research questions:

Research question 1. To what extent do students' competencies on the procedural components of problem solving (i.e., understanding, planning, searching mathematical strategies, and looking back) affect their mathematical modeling competence?

Research question 2. How does the instructional approach (i.e., collaborative learning and problem posing) mediate the impact from students' competencies on the procedural components of problem solving to their mathematical modeling competence?

This study aimed to investigate the structure of mathematical problem solving competence (i.e., the four procedural components and mathematical modeling) and the relationships among the components. Additionally, the current study observed that two main teaching strategies (i.e., collaborative learning and problem posing) mediated the effects of procedural components on students' mathematical modeling competence.

Figure 1 depicts the hypothetical research model of the current study with the following specific hypotheses:

H1. Mathematics problem solving competence consists of four distinguished procedural components and one mathematical modeling competency.

H2. Mathematical modeling is influenced by the other procedural components of mathematical problem solving.

H3. The effects of procedural components of mathematical problem solving on mathematical modeling competence are positive.

H4. Collaborative learning environment plays a role in increasing the synergy effect of mathematical procedural components on mathematical modeling.

H5. Problem posing plays a role in increasing the synergy effect of mathematical procedural components on mathematical modeling.

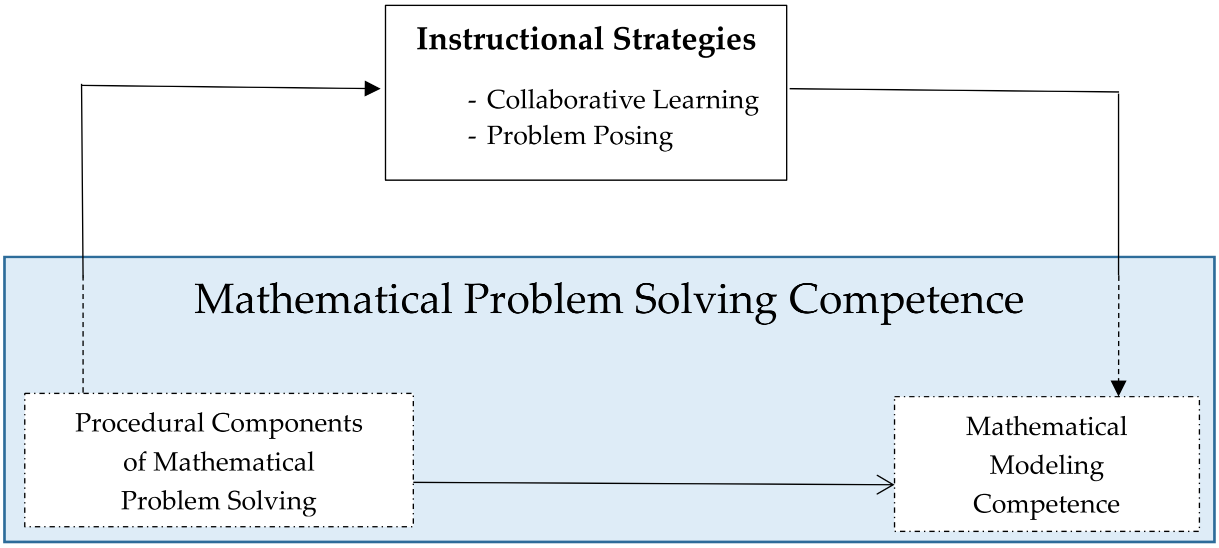

Figure 1. Hypothetical Model

Method

Participants

The participants were students from four regular high schools, one autonomous private high school, and one specialized high school in three big cities, Seoul, Gyeonggi, and Incheon. The selection of the students was conducted through the convenience sampling approach (Etikan, Musa, & Alkassim, 2016). Demographic bias that might rise from this sampling approach was reduced by selecting different school types in Korea. The types of high schools in Korea are classified into regular high schools, autonomous private high schools, and specialized high schools depending on the curriculum contents. A regular high school is a general high school in Korea, and the majority of high schools are regular high schools. An autonomous private high school is one in which the curricula and undergraduate programs are operated autonomously in accordance with the school's founding philosophy. Specialized high schools are secondary schools with enhanced coverage of certain subjects that constitute the specialization of the school. In the second year of high school, Korean students choose one of the following three tracks: humanities, natural sciences, or vocational tracks. Humanities are a track where students study culture, social science, history, and linguistics. Natural science is a track where students study natural order and logic such as science and mathematics. Lastly, the vocational track cultivates the specialists and professionals in specific fields.

A total of 1,224 students were surveyed. The responses of 78 students were excluded from this study as they did not provide proper answers or intentionally gave the same answers for all the questions. In the valid sample (n=1,146), 397 (34.6%) were female and 749 (65.4%) male students. Further, 303 (26.5%), 523 (45.6%), and 320 (27.9%) students were in their first, second, and third years, respectively. Students were asked to report their grades as high, intermediate, or low score reflecting their mathematics grade in the regular school examination of the previous semester. According to their response, 336 (29.3%), 401 (35.0%), and 409 (35.7%) students reported high, intermediate, and low scores, respectively.

From the perspective of the academic course (n=1017), 532 (52.3%) students were in a humanities course and 485 (47.7%) students were in a vocational course. The students of liberal arts, natural science, and vocational study demonstrated 21.8% (222), 30.5% (310), and 47.7% (485) participation rate, respectively.

Procedures

In this study, the measurement development eight step procedure (i.e., materialization of the concept of measurement, development of a preliminary question, determination of measurement type, the preliminary question review, the preliminary investigation and result analysis, and the main investigation and result analysis) in Devellis (2003) were modified and adopted. Prior to the main analysis, a pilot study was conducted to develop the measurement instrument to evaluate the mathematical problem solving competence used in this study. To develop the measurement instrument, existing literature was reviewed to document the concepts and relevant sub-elements of mathematical problem solving competence. In general, competence assessment measurement uses the degree of competence as a scale, which is based on behavioral examples and indicators that describe the targeted competence (Thornton & Rupp, 2006). Adopting an approach by Thornton and Rupp (2006), a survey was administered to students with excellent mathematical problem solving competence to examine the behavioral examples of mathematical problem solving competence, and provisional preparatory questions were deducted. There were 130 survey participants for the pilot study, who were enrolled in four regular high schools in two big cities (Seoul and Incheon) in Korea. There were 28 (21.5%), 68 (52.3%), and 32 (26.2%) students in first, second, and third grades, respectively. They were either winners of the mathematics competition, had received a high grade in the problem solving performance assessment, or were members of a mathematics club. The content validity test was conducted two times, in cooperation with a mathematics education expert and five teachers, to verify the validity of the preparatory questions. Additionally, the face validity test was performed to review the intelligibility of the preparatory questions. Of the 130 students, 32 first-grade students attended the face validity. Finally, 40 six scale items were derived from the preparatory questions. The questionnaire in this study was administered by teachers that were in charge of classes. The teachers explained to students the purpose of the assessment and 1,224 students filled out self-reported questionnaires. Finally, students' responses were analyzed quantitatively using exploratory factor analysis, confirmatory factor analysis, and path analysis.

Measurement

The measurement employed in the study consisted of seven constructs: Understanding, Planning, Strategy Seeking, Looking Back, Mathematical Modeling, Collaborative Learning, and Problem Posing. Detailed explanations for each construct were followed in the next sections.

Understanding. The "Understanding" of a problem was defined as the ability to clarify the problem by comprehending the purpose of the problem, along with the given conditions and information. Based on this definition, the problem understanding section consists of six items. Four of these were developed using literature review, including Kintsch & Greeno (1985) ("Understand the terms used in the question," "Understand the contents and situations after reading the question"), Lee, Chang, Lee, and Park (2003) ("Understand the conditions given in the question"), and Pólya (1957) ("Know what the question is seeking for"). The remaining two items were developed by the researcher to measure if the participant understands the relationships between the given conditions: "Understand the relationships between the given conditions," and "Understand the relationships between the given conditions and what the question is seeking."

Planning. The "Planning" was defined as the ability of properly implementing important mathematical principles or results, and devising plans to obtain the solution. Based on this definition, the planning section consisted of five items. Of this, two items were developed through literature review, including Kim, Park, Choi, and Kim (2011) ("Compare which of the various pieces of information is helpful in resolving the problem"), and Lee et al. (2003) ("Collect information about the same or similar solutions of problems that have been solved before"). One item was developed by the researcher to measure the ability to select a proper solution: "Search for the best solution among various possibilities." Two items were developed through the survey to measure the ability to plan and use the solution method: "Establish a plan of how to resolve the problem," and "Search for the best solution among various possibilities."

Strategy Seeking. "Strategy Seeking" was defined as the ability to explore and utilize various and proper strategies to solve a problem. Based on this definition, the strategy seeking section consisted of five items. Of these, one item was developed by the researcher, and four items were developed from the pilot study to assess the ability to utilize proper strategies: "Establish a detailed equation to find the answer required by the problem," "Try to express the problem in pictures to solve the problem," "Try to draw a figure to solve the problem," "Try to make a table to find what is required in the question," and "Try to draw a graph to solve the problem."

Looking Back. "Plan Execution" and "Looking Back" were defined as the ability to execute the planned solution, and evaluate the solution method and the answer through the process of investigation and self-reflection. Based on this definition, the looking back section consist of four items. One item was developed through literature review of Kim et al. (2011) ("Review the problem solving process after resolving the problem"). One item was developed by the researcher, and two items were developed through the survey to measure the ability to self-examine and assess the solution: "Review and evaluate the problem solving process yourself and look for a better solution," "Check and prove the results of the problem," and "Compare the correct answer with the answer you found."

Mathematical Modeling. The "Mathematical Modeling" was defined as the ability to practically demonstrate and analyze real-life problems in terms of mathematical expressions to draw conclusions and comprehend them based on the situation. Per this definition, the mathematical modeling section consists of seven questions. Three items were developed through literature review, including Son (2007) ("Express various daily-life problems with figures," "Express various daily-life problems with graphs," and "Express various daily-life problems with equations"). Two items were developed by the researcher to measure if the participant is able to mathematically analyze the given situation: "Find problems in the surrounding circumstances and consider mathematical solutions" and "Find mathematical approaches and solutions to every-day problems." Two items were developed from the pilot study to assess the ability to demonstrate various real-life situations in terms of mathematical language: "Express various daily-life problems with pictures," and "Express various daily-life problems with tables."

Collaborative Learning. "Collaborative Learning" was defined as collectively addressing the problem, with a balanced share of responsibility and interaction. Based on this definition, the collaborative learning section consists of six questions. Five items were developed through literature review, including Lee (2013) ("Look for the role you can do in the group to solve the problem, and actively participate," "Discuss the methods to solve the problem with friends," "If multiple methods are suggested, compare each other's opinions and find an optimal method," and "Collect various opinions and find a better way to solve the problem") and Lee et al. (2003) ("Respect other opinions, although they are different from my opinion"). One item was developed by the researcher to assess if the members of a group can cooperate to solve the problem: "Help your friends if they have difficulties solving the problem."

Problem Posing. "Problem Posing" was defined as the ability to transform the given problem or to create new problems in order to solve the given problem. Based on this definition, the problem posing section consists of seven questions. Four of these were developed through literature review, including Brown and Walter (1983) ("Modify the given conditions to create a different question"), Kilpatrick (1987) ("Create a question with the given conditions"), and Na (2001) ("Create a question using pictures" and "Create the right question for the given expression"). One item was developed by the researcher and two were developed using the survey, to measure if the participant is able to modify or expand a given situation: "Apply different situations for the question," "Create a problem with different things to find," and "Integrate multiple questions to create a question."

Data Analysis

Descriptive statistics (i.e., mean, standard deviation) and correlation coefficients of variables were computed and reported (see Table 1). Prior to path analysis, revealing the relationships among the components/factors and covariates of "mathematical problem solving," exploratory and confirmatory factor analysis (EFA and CFA, respectively) were implemented to test construct validity. Based on the results of the EFA analysis, 40 question items and 7 constructs (understanding (UD), planning (PL), strategy seeking (SS), looking back (LB), collaborative learning (CL), mathematical modeling (MM), and problem posing (PP)) were extracted and confirmed. The number of questions for each construct was as follows: six questions for UD, five for PL, five for SS, four for LB, six for CL, seven for MM, and seven for PP. Results showed good internal consistency among all constructs, with Cronbach's alphas ranging from 0.792 to 0.955 (> 0.70), which represent all acceptable values dependent on a commonly accepted rule of Cronbach's alpha (George & Mallery, 2010).

In the current study, composite variables were used in the structural equation modeling analyses because using manifest variables was more likely to reflect the characteristics of the sample rather than of the population (Little, Cunningham, Shahar, & Widaman, 2002; McDonald, Behson, & Seifert, 2005). Based on the CFA results, composite variables were computed as mean values of the items within each construct (i.e., UD, PL, SS, LB, CL, and PP). For the composite variables, descriptive statistics including means, standard deviations, skewness, and kurtosis were computed (Table 1). Means, standard deviations, and correlation coefficients were useful in constructing a proposed structure representing the relationships among the factors of mathematical problem solving and outcome variable (i.e., mathematical problem solving skills/competence). Skewness and kurtosis were used to test normality of each variable. As most variables used in the current study represented skewness and kurtosis values between -2 and 2, it was possible to conclude that the distribution of each composite variable was approximately symmetric and normal (George & Mallery, 2010).

Indirect effects. Indirect effects were estimated using the delta method in Mplus (Muthén & Muthén, 1998-2012). Among the seven constructs, the variables CL and PP were hypothesized to serve as mediators, converting the effects of students' ability on the four different phases (UD, PL, SS, and LB) into students' mathematical problem solving skills. Therefore, direct and indirect paths from students' competence on four separate mathematical problem solving phases to mathematical problem solving skills were estimated and tested for statistical significance.

Table 1. Correlation Coefficients Among the Variables

	1. UD	2. PL	3. SS	4. LB	5. CL	6. PP	7. MM
2. PL	0.649**	—	—	—	—	—	—
3. SS	0.591**	0.585**	—	—	—	—	—
4. LB	0.551**	0.598**	0.431**	—	—	—	—
5. CL	0.566**	0.557**	0.421**	0.547**	—	—	—
6. PP	0.372**	0.475**	0.435**	0.423**	0.355**	—	—
7. MM	0.367**	0.474**	0.521**	0.348**	0.369**	0.685**	—
Mean	4.147	3.896	3.801	3.949	4.203	2.973	3.036
Std. Deviation	1.020	1.023	1.147	0.978	0.948	1.188	1.247
Skewness	-0.311	-0.219	-0.295	-0.207	-0.496	0.149	0.173
Kurtosis	0.108	-0.181	-0.388	-0.030	0.354	-0.434	-0.553

Note. UD = understanding; PL = planning; SS = strategy seeking; LB = looking back; CL = collaborative learning; PP = problem posing; MM = mathematics modeling; **. Correlation is significant at the 0.001 level (2-tailed).

Results

Factor Analysis

To examine the factors of mathematical problem solving competence, factor analyses were employed. The results from the EFA extracted seven constructs of the mathematical problem solving skill (H1). Fit indices indicated that the hypothesized model was fitted into the data well (RMSEA=0.067, CFI=0.932, SRMR=0.020). The seven constructs were named understanding (UD), planning (PL), strategy seeking (SS), looking back (LB), collaborative learning (CL), mathematical modeling (MM), and problem posing (PP). CFAs yielded relevant factor loadings ranging from 0.508 to 0.925 for each construct. To test the model, fit indices of CFAs, RMSEA, CFI, and SRMR were adopted (Brown, 2006). The CFIs of the seven CFAs were higher than 0.939. RMSEAs and SRMRs of the seven CFAs were lower than 0.084 and 0.034. These fit indices reflected that each construct model was a mediocre or acceptable fit for the data (Brown, 2006; Marsh, Hau, Artelt, Baumert, & Peschar, 2006).

Path Analysis

Path analysis model revealed substantial relationships between constructs of students' competence on mathematical problem solving phases, mathematical modeling skills, and instructional strategies (collaborative learning and problem posing). According to the hypotheses, we proposed and tested the final model (see Figure 2). Fit indices (χ^2/df=3.042/1 (not rejected), RMSEA=0.042, CFI=0.999, SRMR=0.006) indicated the model fits into the data well. All path coefficients were statistically significant with the exception of the path from UD to MM (H2 and H3). The paths from PL to MM and from SS to MM were all positive and 0.111 and 0.243, respectively. The path from LB to MM represented negative coefficient ($\beta = -0.059$), which was an unexpected result based on the correlation coefficients and hypothesis 3. Given the correlation coefficient between LB and MM was positive and significant, the path from LB to MM was also expected to be positive. However, the path loading from LB to MM in the path analysis was negative and significant. Therefore, the authors regarded this result as suppressor effect (Arah, 2008; Pearl, 2009).

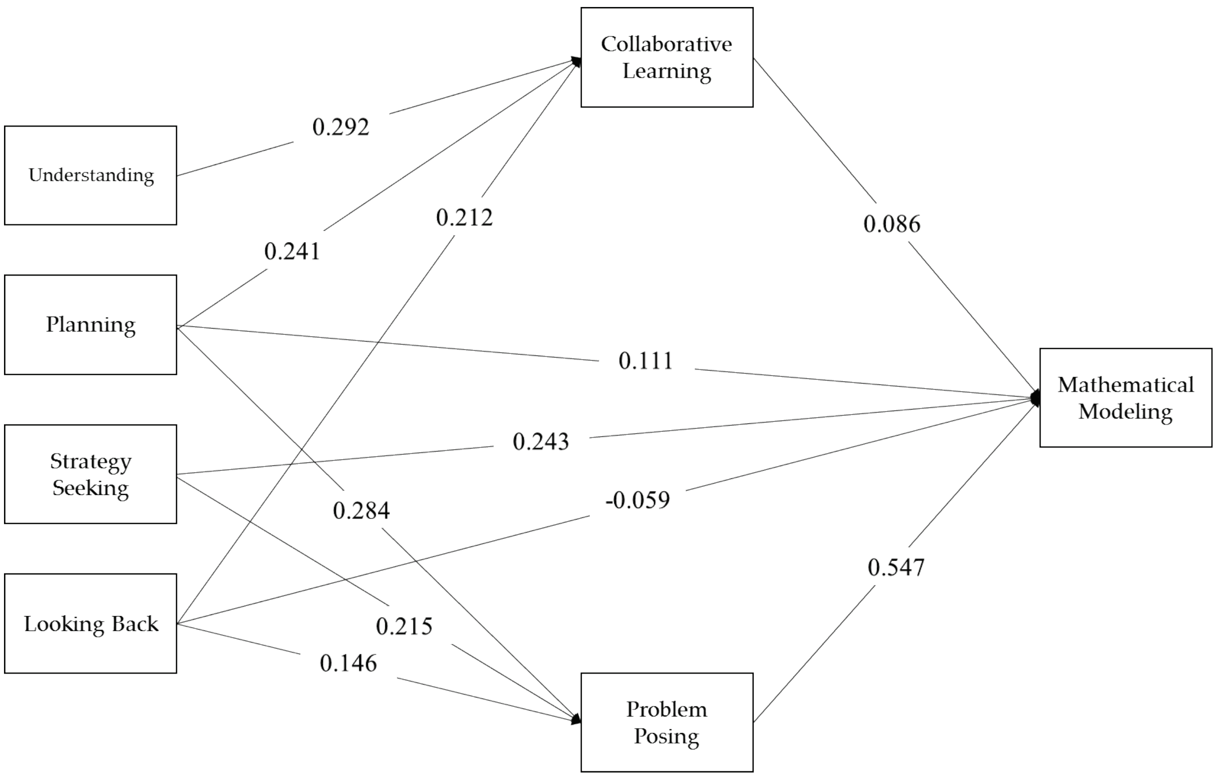

Figure 2. Final Model of the Relationships among Problem Solving Components

Indirect Effects. To examine the mediation effect of instructional strategies (collaborative learning and problem posing), indirect effects from UD, PL, SS, and LB to MM were tested using Mplus delta analysis (see Table 2). The paths from CL to MM and PP to MM were statistically significant and the path from PP to MM presented a high coefficient ($\beta = 0.547$). Among the indirect effects that pass through CL, three indirect effects (UD→CL→MM, PL→CL→MM, and LB→CL→MM) were statistically significant and positive (H4). With regard to the problem-posing strategy, three indirect effects (PL→PP→MM, SS→PP→MM, and LB→PP→MM) were statistically significant and positive (H5).

Table 2. Mediation Effects of Instructional Strategies

Indirect Effects		**Estimates**	***p*-value**
Effects from UD to MM			
	UD→CL→MM	0.025	0.002
	UD→PP→MM	-0.002	0.919
Effects from PL to MM			
	PL→CL→MM	0.021	0.003
	PL→PP→MM	0.156	<0.001
Effects from SS to MM			
	SS→CL→MM	0.118	<0.001
	SS→PP→MM	0.243	<0.001
Effects from LB to MM			
	LB→CL→MM	0.018	0.003
	LB→PP→MM	0.080	<0.001

Note. UD = understanding; PL = planning; SS = strategy seeking; LB = looking back; CL = collaborative learning; PP = problem posing; MM = mathematics modeling.

Discussion and Conclusion

The current study investigates the procedural components of mathematical problem solving and their impacts on mathematical modeling competence. Mathematical problem solving and mathematical modeling are the critical competencies that are documented as main goals to be pursued in the present curriculum of mathematics education. Therefore, it is important to find instructional strategies to improve students' mathematical problem solving and modeling competencies. Consequently, the findings of this study will contribute to the mathematics education field by providing mathematics educators and teachers the information drawn from the analyses investigating the relationships between mathematical problem solving procedural components, mathematical modeling, and instructional strategies such as collaborative learning and problem posing.

The findings of the study suggest that procedural components of mathematical problem solving had a varied impact on mathematical modeling competence. A previous study by Schukajlow et al. (2015) examined that scaffolding with a solution plan (consisting of understanding task, searching mathematics, using mathematics, and explaining results) supports the development of students' modeling competency. With findings similar to those by Schukajlow et al. (2015), the current study also supports the positive impact of recognizing problem solution steps when exploring mathematical modeling. However, that study does not investigate *to what extent* each phase of the solution plan affects the development of students' mathematical modeling competence. In the present study, authors resolved this limitation by revealing the structure of the mathematical problem solving, procedural components and mathematical modeling competence.

The findings from the current study are meaningful in regards to clarifying the relationships between the procedural components of mathematical problem solving and mathematical modeling competency. According to the direct effects from the procedural components of mathematical problem solving to mathematical modeling competence, the four phases of mathematical problem solving showed varied impacts on mathematical modeling competence. For example, understanding a problem had no direct effect on mathematical modeling competence. This result might come from the difference between mathematical problem solving and modeling. According to Pollak (2012), the mathematical modeling process includes problem finding as well as problem solving. In addition, mathematical modeling begins in the real world whereas problem solving begins with an idealized real-world. That is, students that understand a mathematical problem might have difficulties identifying a problem in the real world situation and applying mathematical modeling. For this reason, the effect from understanding a problem on mathematical modeling might be statistically insignificant. The phases of planning and seeking strategies showed positive impacts on developing students' mathematical modeling competence. However, these findings do not imply that the phases of understanding are not meaningful in the process of mathematical modeling and solving a mathematical problem. Instead, they indicate that there is a difference in the degree of significance of each procedural component of mathematical problem solving for mathematical modeling. In mathematical modeling, seeking mathematical strategies was more critical than other procedural components such as understanding and planning.

Given that the indirect effects of procedural components of mathematical problem solving on mathematical modeling were statistically significant, students' competence of mathematical problem solving was closely related to their mathematical modeling competence. When interpreting this finding, it is necessary to remember that the four procedural components were more likely for purely mathematical problem solving. Whereas, the definition of mathematical modeling clearly indicates that mathematical modeling is applied to find solutions for *real-world* problems with mathematics (Pollak, 2012). From this perspective, the result that the direct and indirect paths from UD to MM showed different statistically significance can be rationally interpreted. That is, students' ability to understand real-world problems is at a different level from the ability to understand pure mathematics problems (Cirillo et al., 2016). The procedure of mathematical modeling is constituted within the phase of connecting and interpreting real-world problems with mathematical tools (Blum & Ferri, 2016), which is omitted when solving pure mathematics problems. At the same time, understanding the terminology used in mathematics problems–what is known/unknown, and what are the constraints–was also needed in the process of setting a problem in the real-world problem situation because collaborative learning environment was observed to mediate students' ability to understand mathematics problems for mathematical modeling competence.

The role of problem posing in indirect effects is also worth noting. The results from the study show that the coefficient from PP to MM is high, indicating that an instructional approach such as problem posing can positively influence students' mathematical modeling competence. The positive role of problem posing for mathematical modeling has been pointed out by many researchers in the past (English, 1997; Lavy & Bershadsky, 2003; Lowrie, 2002; NCTM, 2000), and has been confirmed once again through the results of this study. In addition, the mediation effects of PP on the paths from PL to MM and SS to MM were higher compared to other mediation effects. This finding indicates that instructional strategies using problem posing may be more effective for procedural components such as planning and searching for strategies than for other components such as understanding and looking back. Very limited research has been done to determine if problem posing is particularly effective on certain components of problem solving that were explored by the current study.

The findings of this study might be due to the instrument that was used, which measured competence, and not an ability or skill. The concept of competence was deemed as multifaceted, including affective resources as well as cognitive skills. For this reason, it was assumed that the instructional strategies (collaborative learning and problem posing) were more likely to affect students' development of competencies rather than abilities or skills because affective and cognitive components constitute mathematical competence. A literature review suggests that collaborative learning and problem posing strategies (Crespo & Sinclair, 2008; DiDonato, 2013; Lester, 2013; NCTM, 1989; Yuan & Sriraman, 2011) had positive impacts on students' affective factors, which in turn positively influenced students' mathematical competence.

Certain mathematical competencies may affect other competencies (Albaladejo et al., 2015). The current study contributes to this assertion with concrete results from experimental research. In general, the results of the study support the premise that the competence of problem solving positively affects the development of mathematical modeling competence. Moreover, the investigation on the relationships among the procedural components of mathematical problem solving and mathematical modeling competencies exhibits *how* the mathematical problem solving competence influences mathematical modeling competence.

The findings of the study have several implications for mathematics teachers and educational policy makers. First, the study suggests several instructional strategies to develop students' mathematical modeling competence. Specifically, collaborative learning and problem posing in mathematics classrooms were shown as mediators producing synergy between mathematical problem solving and modeling competencies. Second, the mediation effects of collaborative learning and problem posing reveal that a teaching-learning approach involving these instructional strategies has a positive effect on the development of students' mathematical modeling competence. For example, project-based learning is usually implemented in small groups, which was examined as an appropriate instructional approach improving students' problem solving competence. Finally, the survey instrument employed in the study can be used as an assessment tool for measuring students' mathematical problem solving and modeling competencies.

Despite the significant theoretical and practical contributions of this study, it has several limitations that require attention. First, the participants were limited to Korean students in this study. Students from other countries may have different structural constructs for the procedural components of problem solving and mathematical modeling competencies. Therefore, before adopting the instructional strategies suggested in this study to improve mathematical modeling competence for students in other countries, it will be necessary to examine the mediation effects in advance. Second, the instrument used for investigating the students' mathematical problem solving and modeling competencies was self-reported. Even though self-reported variables may depend upon students' subjective judgment, the current survey instrument intended to guarantee objective criteria by including observable behavioral characteristics into the items. However, the authors strongly suggest developing another instrument using teacher-report or peer-report approach and comparing the results with the current findings in future studies.

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