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Metacognitive Experiences: Mediating the Relationship between Metacognitive Knowledge and Problem Solving *

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Abstract

This study examined the mediating role of metacognitive experiences in the relationship between metacognitive knowledge and mathematical problem solving performance. The mediating effect of metacognitive experiences in the hypothesized model was tested through the latent variable structural equation modeling statistical analysis technique. The proposed structural model of problem solving was tested with the data obtained by using convenience sampling method from 406 eight-grade students. The results indicated that students' task related metacognitive experiences have a significant mediating effect on the relationship between metacognitive knowledge and problem solving performances. The results are important for two reasons. Firstly, it addresses the assessment of metacognition by presenting the important role of metacognitive experiences in governing the relationship between metacognitive knowledge and actual performance. Secondly, the model analysis was conducted just over a single math problem. The use of metacognitive experiences as an online single-problem-oriented self-report metacognitive assessment tool might pave the way for easy assessment of student's metacognitive functioning.

Keywords

Metacognition
Metacognitive experiences
Metacognitive knowledge
Mathematical problem solving
Mediation analysis
Structural equation modeling

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Introduction

The importance of metacognition in problem solving performance has been highlighted in many studies (Artz & Armour-Thomas, 1992; Aşık, 2015; Desoete & Veenman, 2006; Mihalca, Mengelkamp, & Schnotz, 2017). Significant positive effects of metacognition on problem solving performance are observed in many studies (Jacobse & Harskamps, 2012; Kramarski & Mevarech, 1997), especially when metacognition is assessed through observations and think-aloud protocols. Nevertheless, it is not as easy to draw the same conclusion if self-report instruments are used to assess students' metacognitive knowledge and regulation. Many studies failed to show significant direct effects of metacognitive knowledge on problem solving (Depaepe, De Corte, & Verschaffel, 2010; Kuyper, Van der Werf, & Lubbers, 2000; Malpass, O'neil, & Hocevar, 1999; Schraw, 1997). The scarcity of foreseen evidence for the role of metacognition on problem solving performance led us to include the mediating factor in our investigation to offer a meaningful explanation to the relationship between

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metacognitive knowledge and performance. Student's task related metacognitive experiences conceptualized as online state metacognitive knowledge came into prominence at this point.

Although there are many studies focusing on the relationship between metacognition and problem solving performance, studies investigating simultaneous assessment of general and task specific metacognition are scarce. This led us to propose and test a structural model, which tries to offer meaningful insight into the role of metacognition on mathematical problem solving. In the present study, we specifically examined whether metacognitive experiences function as a mediator between metacognitive knowledge and problem solving performance. It is imperative that we better understand the effect of students' metacognitive knowledge and learn whether there is an indirect effect of metacognitive knowledge through metacognitive experiences on problem solving performance. Structural equation modeling (SEM) analysis was used to assess the mediator effect and indirect effect of metacognitive experience on problem solving.

Metacognition and Problem Solving

Problem solving has been emphasized as a central focus in school mathematics (Lesh & Zawojewski, 2007; Lester & Kehle, 2003; National Council of Teachers of Mathematics, 2000; Polya, 1957; Schoenfeld, 1989, 1992). On the one hand, it is defined as a way to teach all mathematics topics; on the other hand, problem solving becomes the main aim of life-long learning. Many students have difficulties in solving word problems even when they think that the problems are easy to solve (Campione, Brown, & Connell, 1989; Cummins, 1992; Hegarty, Mayer, & Monk, 1995; Koedinger & Anderson, 1998). Good problem solvers are able to plan, monitor, evaluate and regulate their problem solving processes more consistently compared to poor problem solvers (Derry, 1989; Sarver, 2006; Van de Walle, 2003). Having high degrees of control over these steps indicates the existence of metacognition, which is a distinctive characteristic of successful problem solvers (Hong, 1995; Mayer, 1998; Schoenfeld, 1985).

Metacognition has a role in mathematical problem solving especially when students build an appropriate representation of the problem and check the results of the calculations (Schoenfeld, 1983; Verschaffel, 1999). It denotes better performance for students in identifying and working on the problem strategically (Davidson & Sternberg, 1998), allows students to use their acquired knowledge and procedures in a meaningful and effective way (Lucangeli, Tressoldi, & Cendron, 1998) following the leading role of cognitive capacity and skills (Carr & Jessup, 1995). Students benefit from metacognitive components such as when and how to use the required skills, when previous knowledge and gained skills for successful problem solving are emphasized in instruction (Jiang, Ma, & Gao, 2016; Lucangeli & Cornoldi, 1997; Mayer, 1998; Van der Stel, Veenman, Deelen, & Haenen, 2010).

The definitions of metacognition (Costa, 1984; Flavell, 1979; Mevarech, Tabuk, & Sinai, 2006) have been mostly conceptualized around two processes: knowledge about one's own cognition and regulation of cognitive activities. Metacognitive skills, metacognitive knowledge and metacognitive experiences constitute the three facets of metacognition. Metacognitive knowledge refers to a person's declarative knowledge about the interactions between the person, task and strategy, while metacognitive skills refer to procedures and behaviors employed to regulate cognitive activities (Flavell, 1979). Metacognitive knowledge is briefly identified as someone's stored world knowledge. On the other hand, metacognitive experiences, also referred to as self-initiated metacognition, were defined as any conscious cognitive and affective experience emerging in any kind of task processing (Efklides, 2001; Flavell, 1979).

Metacognitive experiences are not knowledge or mastery that one has gained from their past experiences. They do not come from the sources related to or out of the person's own experiences (Efklides, 2001). Functioning as the interface between the self and the task, metacognitive experiences reflect present subjective experiences that arise as a result of a cognitive activity (Efklides, 2002; Gombert, 1993). As the products of the working memory, metacognitive experiences are closer to actual cognitive processing and they monitor online processes of cognition (Efklides, 2001, 2006a). They have close relationships with cognitive, motivational and metacognitive knowledge and in turn, they are influenced by these factors. Metacognitive feelings, online metacognitive knowledge (also referred to as online task-specific knowledge) and metacognitive judgments are the forms where metacognitive experiences might be observed (Efklides, 2006b, 2009).

Metacognitive experiences serve the monitoring and control of the learning process. They help students to control and direct their present and future behaviors by integrating information about the self and the experiences (Efklides, 2009). Metacognitive experiences at the prospective phase, which are measured first as soon as a task is presented and before the person starts working on it, are feelings of familiarity, feelings of understanding, feelings of difficulty, estimate of effort and predicted solution correctness (Efklides, 2006a). They provide an intrinsic context and future motivation for learning (Desoete & Veenman, 2006). Efklides, Kiorpelidou, and Kiosseoglou (2006) underlined the significant role of present metacognitive experiences while working on a task, as opposed to metacognitive knowledge about such experiences or relevant information from the past.

Despite the consensus that all three facets of metacognition have important roles in learning, metacognitive experiences is the area least focused on by researchers (Efklides, 2009). There are many research studies focusing on metacognitive processes in mathematical problem solving which have been carried out in the past two decades (Aşık, 2015; Desoete, Roeyers, & Buysse, 2001; Desoete & Roeyers, 2006; Downing, Kwong, Chan, Lam, & Downing, 2009; Mevarech & Fridkin, 2006; Ozsoy & Ataman, 2009; Özcan & Erktin, 2015; Schoenfeld, 1987). On the other hand, metacognitive experiences have not been examined deeply in the problem solving context (Efklides, 2009; Tornare, Czajkowski, & Pons; 2015). A few studies have been conducted in the related field and most of them demonstrated positive relationships with problem-solving performance (Lucangeli, Coi, & Bosco, 1997). Investigating the role of metacognitive experiences in problem solving may provide a clear explicit understanding of mechanisms in the nature of metacognition.

Metacognition researchers have frequently focused on metacognitive skills such as planning, self-checking, cognitive strategy use and evaluation processes (Ku & Ho, 2010; O'Neil & Abedi, 1996; Schraw & Denison, 1994; Van der Stel et al., 2010) and metacognitive knowledge (Çetinkaya & Erktin, 2002; Hong, O'Neil, & Feldon, 2005; Schraw, 1997; Tobias & Everson, 2002; Wilson & Bai, 2010). Many studies have demonstrated positive correlations between metacognition skills and problem solving performance (Lucangeli et al., 1998; Mevarech & Amrany, 2008; Özcan & Erktin, 2015; Rysz, 2004). Nevertheless, some failed to show the role of metacognitive knowledge on problem solving performance (Ader, 2004; Depaepe et al., 2010; Hong & Peng, 2008; Kuyper et al., 2000; Schraw, 1997). Two issues, namely the conceptualization of metacognition and the difficulty of assessment, have been discussed as the possible roots of these contradictory findings (Artz & Armour-Thomas, 1992; Desoete & Roeyers, 2006; Veenman, Van Hout-Walters, & Afflerbach, 2006). They are relevant discussion issues for researchers interested in metacognition. Studies focusing on this broad context would enrich our understanding.

Two major arguments have been the object of interest for metacognition researchers. The first is about its conceptualization and the second is about its assessment (Veenman et al., 2006). The conceptualization of metacognition leads to a major question of whether it is a cognitive trait or a situation specific construct and to what degree metacognition can be generalized. Traits are considered to be lasting characteristics of individuals. Although state characteristics are highly predictable by traits, they are relatively changeable depending on the environment or occasion (Spielberger, 1975). Besides, studies focusing on multiple tasks rather than a specific task or a domain reveal questionable results. Although literature points out that metacognition is significantly correlated with academic performance in both state and trait situations (Hong, 1995; Hong & Peng, 2008; Hong et al., 2005; O'Neil & Abedi, 1996), how metacognition, perceived as a general characteristic (trait), is related to task specific (state) metacognition is an important issue that needs to be clarified for educators.

Another argument is over the assessment of metacognition (Veenman, Bavelaar, De Wolf, & Van Haaren, 2014). Many methods are being used for the assessment of metacognition, but which method assesses what we focus on more precisely might be a burning question in the field. In this respect, online versus off-line methods have been discussed intensively (Veenman, 2005). Off-line methods include students' self-reports resulting from their prior experiences. On the other hand, online methods are based on actual performance of students while working on a task (Saraç & Karakelle, 2012; Veenman et al., 2014). Even though online methods seem to be a better predictor of performance, off-line methods are more frequently used due to the fact that they are easy to administer and analyze (Veenman, 2013). In this regard, the strengths of online and offline methods for the prediction of outcome measures need to be clarified. Thus, multi-method design research studies could be useful to understand the difference between various assessment tools (Veenman, 2011b; Veenman et al., 2006).

Although trait self-report questionnaires are a way to assess students' metacognitive knowledge, students' actual metacognitive engagements might go beyond what trait self-reported instruments tell us (Schellings & van Hout-Walters, 2011; Veenman, 2011b, 2015). Metacognitive experiences as single-item self-report online measures may be much more explanatory to examine deeply the role of metacognition in a problem solving situation (Mihalca et al., 2017). Given the importance assigned to metacognitive experiences, it is possible to examine students' online cognitive activity during task processing. Information is needed as to the relationships between macro-level measures such as metacognitive knowledge that focus on more general trait characteristics of individuals, and micro-level measures such as metacognitive experiences operating as state measures in problem solving situations.

This study sets out to explain the role of metacognition on math problem solving performance by focusing deeply on students' metacognitive knowledge and metacognitive experiences while solving math problems. The mediating effect of metacognitive experiences on the relationship between metacognitive knowledge and mathematical problem solving performance of eight graders was tested. In this research, student's metacognitive knowledge was considered as a task independent trait characteristic and was assessed through an off-line self-report questionnaire. Student's task related metacognitive experiences were investigated through online state measures. In many former studies, problem solving performance were assessed through achievement tests and exam scores. Tests including a large number of items may not reflect a student's actual performance on a single problem. Assuming that the best measurement of self-regulated strategies could be done in domain specific contents such as a specific task (Pintrich, 2004), it was thought that analyzing what students go through in solving only one mathematics problem would lead to a better understanding of the purposes set out in this study. The model was tested on three math problems. Analyses were conducted separately for each mathematics problem. Figure 1 demonstrates the hypothesized model to be tested in the present study.

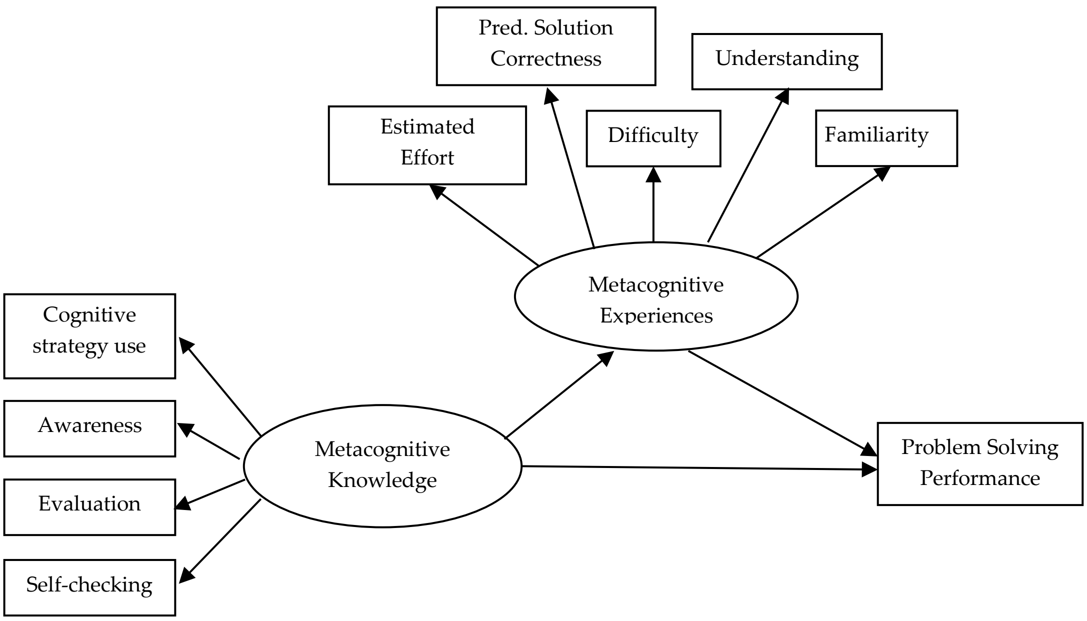

Figure 1. The Hypothesized Model

In order to test the hypothesized mediation model, the following questions were investigated in the path relations:

1. Is metacognitive experiences a mediator variable in the relationship between metacognitive knowledge and problem solving performance?
   a. Does metacognitive knowledge as an independent variable have significant direct effects on both metacognitive experiences (mediator) and the problem solving performance (dependent variable)?
   b. Do metacognitive experiences have a significant direct effect on problem solving performance?
   c. Does the causal relationship between metacognitive knowledge and problem solving performance decrease substantially upon the addition of metacognitive experiences (mediator) as a predictor of the problem solving performance?

Method

In order to test the proposed model describing metacognitive experiences as the mediator between metacognitive skills and problem solving performance, structural equation modeling analyses (SEM) were executed. Structural equation modelling (SEM) is a statistical approach used to investigate theory-derived causal hypotheses (Sumer, 2000). Since the proposed model of the study is measured by the unobserved variables, latent variable structural equation modeling method was used in the study. Data were collected on three different measures.

Participants

Structural Equation Modeling (SEM) usually requires large samples of n = 200 in order to test the hypothesized relationships in a meaningful way with fewer measurement errors (Kline, 2011). Therefore, the sample of the study consisted of 406 eighth-grade students from three public (63%, n=203), and two private (37%, n=153) middle schools in Istanbul, Turkey. Of these students, 48 percent were female (n=195) and 52 percent were male (n= 211). The average age of the students was 14. Students in the private schools have to pay a certain amount of tuition fee every year. Public schools are state-financed and therefore are free of charge. Attending a public or a private school may indicate differences in socio-economic status.

Measures

Three different measures were separately used to assess students' trait metacognitive knowledge, prospective metacognitive experiences and problem solving performance.

Metacognitive Skills Inventory: Metacognition was assessed through the Metacognitive Skills Inventory [MSI] (Çetinkaya & Erktin, 2002), which is a self-report instrument for measuring students' trait metacognitive knowledge on their metacognitive skills. The scale comprised of four dimensions: self-checking, awareness, cognitive strategy use and evaluation. There were 32 items in the scale and all items were in a four-point Likert-type format, scaling from 1 (never) to 4 (always). Low scores indicated low levels of metacognitive skill acquisition. It is assumed to be relatively stable over time. The following items are examples of the domains. *Self-checking* (Item 2): I check my work while I am answering a question. *Awareness* (Item 9): I am a good judge of how well I understand something. *Cognitive Strategy Use* (Item 25): I know when each strategy I use will be most effective. *Evaluation* (Item 4): I realize and correct my errors in the tests.

The instrument indicated a high degree of internal consistency with an overall reliability coefficient of .87 in a previous study. Beside judgmental ratings, the factor analysis pointed out overlapping domains and factors as evidence of construct validity (Çetinkaya & Erktin, 2002). Before the present study, a pilot study was conducted with 91 students in two public schools to assess the psychometric properties of the questionnaire. Acceptable internal consistency (.92) was obtained with the corrected item total correlations ranging between .31 and .66.

Metacognitive Experiences Scale: Metacognitive Experiences Scale (Efklides et al., 2006) was translated and used to assess student's online metacognitive feelings and judgments on mathematical word problems. The scale consists of five items and focuses on reflecting students' feelings of understanding, feelings of familiarity, feelings of difficulty, estimated effort, and predicted solution correctness (Aşık, 2009).

Metacognitive experiences can be assessed prospectively and retrospectively, depending on the purpose of the study (Efklides, 2001; Efklides et al., 2006). In this study, students' prospective metacognitive experiences were evaluated in the problem solving process. The following questions were used to measure students' metacognitive experiences on the given tasks: "How familiar is the problem to you?"; "How well do you understand what it requires you to do?"; "How difficult do you feel the problem is?"; "How much effort do you think you have to exert in order to solve the problem?"; "How correctly do you think you can solve the problem?". Participants responded to each question by rating a four-point scale 1 for "*not at all*", 2 for "*a little*", 3 for "*quite*", and 4 for "*very much*".

The scale was adapted into Turkish and later the coherence and appropriateness of the questions for the Turkish language were checked by two experts working as linguists in the field of education. Satisfactory internal consistency estimate for reliability (.78) was also obtained for the scale (Aşık, 2009).

Math Problem Solving Test: Having a complex process of problem solving, rather than an automated and memorized progression process, allows individuals to have more metacognitive involvement (Lucangeli & Cornoldi, 1997; Aşık, 2015). At the same time, purely routine solutions or extremely complex problems lead to reliability and validity problems in the measurement of metacognition (Veenman, 2015). In this context, it has been noted that the mathematical problems to be used in this research cannot be solved only by repeating the learned algorithms. Besides, these problems need to be those that can be defined as daily life problems and do not cover unfamiliar information.

Three problems were asked to assess the participants' performance in problem solving. The first and the third problems were originally taken from the 7th grade math textbook, which was used as a textbook by all students in all participating schools a year before the implementation. The first two of the problems required knowledge of algebra and the third one could be solved using arithmetical operations. The problems were as follows:

- Merve wants to buy a bicycle by saving some money. To buy a bicycle of 150 TL, she has to add 30 TL more than twice the money she has already saved.
    - How much money has Merve already saved?
    - If Merve adds 2.50 TL per day to the money she has already saved, after how many days can she buy the bicycle?
- The number of girls in a class is 13 less than ¾ of the actual size of the class. If the number of girls is 20, then how many boys are there?
- Mr. Bonomo's monthly salary is 1200 TL. He spends 40% of his salary on rent, 35% on supermarket shopping, and 15% for other expenses and saves the rest. At the end of 1 year, he buys a television that costs 580 TL and goes on holiday with the remaining amount. How much money does Mr. Bonomo have for this holiday?

Each problem was scored based on the Holistic Scoring Rubric (Aschbacher, Koency, & Schacter, 1995). In the rubric, student's performance in each problem was evaluated based on five score levels (0 to 4). The highest score of 4 represented an entirely correct answer while the lowest score of 0 was regarded as no answer. The sample scoring criteria corresponding to each score are provided below:

0 point : The student response only repeats information in the problem task.
1 point : The student did consider a constraint or variable of the problem situation.
2 point : The student did not carry the procedures/strategies far enough to reach a solution.
3 point : The student has considered an irrelevant variable or failed to consider a relevant variable.
4 point : The solution and all relevant work is correct.

In order to check the criterion-related validity of the test, correlations between students' performances in math problem solving and their grades at the end of the first semester were calculated. The Pearson correlation coefficient was found to be .73, significant at the .01 level indicating evidence for validity of the problem solving test. The correlation coefficients between each problem were also around .55 ($p < .01$).

Data Collection and Analysis

Initially data on students' metacognitive knowledge was gathered through the Metacognitive Skill Inventory. Secondly, metacognitive experience scale was implemented along with the math problem solving test. As soon as a math problem was presented to the students, they were asked to rate their feelings about the problem on the scale. Later on, they were allowed to solve the given problem. The process was repeated for all three problems.

This study primarily addresses a mediation analysis. Mediation assumes both causality and ordering among the variables. In a causal relationship, variables can be both causes and effects. The standard regression method is not compatible with modeling a mediating relationship, since each variable can be either the cause or the effect in standard regression, whereas the mediator is both the cause and the effect at the same time (MacKinnon & Fairchild, 2009). A standard regression analysis would provide a limited test of the mediational hypothesis (Baron & Kenny, 1986). Thus, structural equation modelling (SEM) analysis was used to provide a more appropriate inference framework for mediation analysis (Gunzler, Chen, Wu, & Zhang, 2013). Using SEM allows us to test the theoretical propositions regarding how metacognitive constructs are theoretically linked to each other.

For the data analysis, the maximum likelihood estimation method was used in SPSS-AMOS 25 structural equation modeling program. At first, a confirmatory factor analysis was carried out on the main sample to assess the measurement model to determine if selected items had significant factor loadings on the latent constructs. Later on, the hypothesized structural model was tested with the data.

In their seminal paper, Baron and Kenny (1986) defined the mediator as a variable to the extent that a variable accounts for the relationship between the predictor and the outcome variables. In our model, metacognitive experiences seemed to function as a mediator between metacognitive knowledge and problem solving performance. In order to claim that mediation occurs in our model, the following conditions must hold (Baron & Kenny, 1986; Hayes, 2009): metacognitive knowledge as an independent variable must significantly affect both metacognitive experiences (mediator) and the problem solving performance (dependent variable); and also metacognitive experiences must have a significant effect on problem solving performance. As a last requirement, the causal relationship between metacognitive knowledge and problem solving performance must decrease substantially upon the addition of metacognitive experiences (mediator) as a predictor of the problem solving performance.

The likelihood ratio test statistics are sensitive to the sample size, and large samples (over 200) may provide unrealistically large chi-square values. In some cases, this may lead to problems of fit (Blunch, 2008; Kline, 2011). In the test for the model fit, rather than probability values, the ratio of chi-square to the degrees of freedom (χ^2/df) value was considered in the first place (Byrne 2001). If the chi-square ratio is equal to or less than three, it indicates an acceptable fit between the hypothetical model and the sample data (Carmines & McIver, 1981).

Various fit indices such as Root-Mean-Square Error of Approximation (RMSEA), Standardized Root Mean Square Residual (SRMR), Non-normed Fit Index (NNFI), Adjusted Goodness-of-Fit Index (AGFI), and Comparative Fit Index (CFI) were evaluated in the present study (Bentler, 1990; Blunch, 2008; Bollen, 1990; Brown & Cudeck, 1993; Gerbing & Anderson, 1993; Hu & Bentler, 1995; Joreskog, 1993). RMSEA and SRMR around .05 is considered a sign of good fit while values below .08 are generally considered satisfying. For a good model data fit, the other expected values should be as follows: AGFI value is above .90; and CFI and NNFI values are close to .95, although values higher than .90 are originally considered representative of a well-fitting model (Blunch, 2008; Byrne, 2001).

Table 1. Means, Standard Deviations and Standard Errors for the Variables (*N*=406)

		Problem 1			**Problem 2**			**Problem 3**		
		$\bar{x}$	*SD*	$SE_{\bar{x}}$	$\bar{x}$	*SD*	$SE_{\bar{x}}$	$\bar{x}$	*SD*	$SE_{\bar{x}}$
State Measures **Metacognitive Experiences**	Problem Solving Performance	1.77	1.34	.07	1.87	1.80	.09	2.06	1.90	.09
	Feeling of Understanding	3.42	.73	.03	3.41	.72	.03	3.25	.82	.04
	Feeling of Familiarity	2.98	.85	.04	3.32	.77	.04	2.93	.85	.04
	Feeling of Difficulty	1.68	.64	.03	1.81	.75	.04	1.97	.72	.03
	Estimated Effort	2.03	.65	.03	2.01	.71	.04	2.24	.72	.03
	Predicted Sol. Correctness	3.20	.80	.04	3.19	.81	.04	3.04	.84	.04

		$\bar{x}$	*SD*	$SE_{\bar{x}}$
Trait Measures **Metacog. Knowledge**	Self-checking	30.13	5.26	.26
	Cognitive Strategy Use	18.31	3.27	.16
	Evaluation	21.86	4.47	.22
	Awareness	25	4.13	.20

Since metacognitive experiences are task-sensitive, the analyses were conducted for all three problems separately. In this report, the results were depicted by taking the first problem into account. At the end of the analysis section, the results for the other two problems were also reported. Similar results were obtained for all three problems.

Results

Descriptive Statistics

Means and standard deviations of the problem solving performance and metacognitive experience scores are shown in Table 1. Since metacognitive experiences are state measures and depend on the given problem; means and standard deviations were given separately for each problem. High scores indicated positive views on metacognitive experiences. On the other hand, they implied high performance in problem solving performance. 4 was the top score for all metacognitive experiences and problem solving performance. The means for the algebra problems, the first two problems, were calculated as 1.77 (SD= 1.34) and 1.87 (SD= 1.8); they were close to each other and there was no statistically significant difference at .01 level. The mean for the arithmetic problem (2.06, SD= 1.9) was significantly higher than that for the algebra problems ($p < .05$). As for the standard deviations, the first problem had a lower one, with 1.34, compared to the others. The mean and standard deviation scores of metacognitive experiences and trait metacognitive knowledge were also listed in the table.

Measurement Model

The measurement model describes the connections between latent variables and their manifest (observed) variables. In this study, the measurement model was determined by taking two latent variables into account: metacognitive knowledge and metacognitive experiences. Confirmatory factor analysis was performed to test the measurement model. In the analysis, each indicator was constrained to load one and only one factor that was specified to measure the model relationships. The chi square value was calculated as $x^2(25) = 56.6$, with a chi-square to degrees of freedom ratio= 2.26. Other indices were revealed as follows: RMSEA= .06, SRMR= .03, AGFI= .95, CFI= .98. Goodness-of-fit indices demonstrated that the measurement model fit the data well. The indices were considered satisfying to treat the item groups as distinct latent variables in the structural equation model.

Table 2. Standardized Factor Loadings and Variances for Each Indicator

		Problem 1		Problem 2		Problem 3	
		Factor Load	R^2	Factor Load	R^2	Factor Load	R^2
Üstbilişsel Deneyimler	Feeling of Understanding	.80	.63	.77	.60	.80	.64
	Feeling of Familiarity	.43	.18	.38	.14	.34	.12
	Feeling of Difficulty	.65	.42	.64	.41	.71	.50
	Estimated Effort	.59	.35	.61	.37	.56	.32
	Predicted Sol. Correctness	.78	.60	.78	.61	.80	.63

		Factor Load	R^2
Üstbiliş Bilgisi	Cognitive Strategy Use	.87	.76
	Self-checking	.84	.70
	Evaluation	.79	.62
	Awareness	.85	.73

Table 2 shows the standardized factor loadings and variance for each indicator. All factor loadings of the indicators for the first problem were statistically significant ranging from .43 to .80. Standardized factor loadings and variance for each indicator of individual tasks respectively are displayed in Table 2. Feeling of familiarity had the lowest loading (.43) on the metacognitive experiences factor and feeling of understanding loaded highest (.80) on the same construct. The indicators of metacognitive knowledge had higher loadings ranging from .79 to .87 with the highest one being cognitive strategy use (.87). Variances of the indicators were calculated in a wide-range from .18 to .76. The explained variances for the indicators of metacognitive experiences were smaller than those of metacognitive knowledge. The smallest variance (.18) was associated with the feeling of familiarity, while the highest (.76) was for the cognitive strategy use.

Table 3 demonstrates the estimated correlations among the first-order factors. The findings showed that all main variables had significant moderate correlations ($p< .01$) with each other ranging from .40 to .55. The significant correlations among first-order constructs were accepted as justification to use the related variables in the hypothesized model for the data to be tested on.

Table 3. The Correlation Coefficients between All Involved First-Order Constructs

	Metacognitive Knowledge	Metacognitive Experiences	Problem Solving Performance
Metacognitive Knowledge	1	.55*	.40*
Metacognitive Experiences		1	.53*
Problem Solving Performance			1

* Correlations are statistically significant at .01 level (2-tailed).

Structural Model

Although students were assessed at three different problems, in order to derive a single direct, indirect and total effect in the model, this part was constrained to report the results by taking only one problem into account. The results indicated that the model had a chi-square value of 67.82 with 32 degrees of freedom ($p<.001$). The chi-square ratio was calculated as 2.12, which means an acceptable fit due to being smaller than 3. Goodness-of-fit indices for the model were calculated as AGFI= .95, CFI= .98, NNFI= .97, SRMR= .03 and RMSEA= .05. The chi-square ratio and fit indices showed that the hypothesized model fit the data well.

Figure 2 displays the final latent variable structural model that was tested for the first problem. Standardized regression (β) and squared correlation coefficients (R^2) were presented on the model. Standardized estimates were reported rather than raw estimates to simplify interpretation. The values corresponding to each path represent the standardized regression coefficients while the ones corresponding to the endogenous variables represent the estimates of squared multiple correlations. The results of the path analysis indicated that all causal relations were significant. The amount of variance was explained by the predictors.

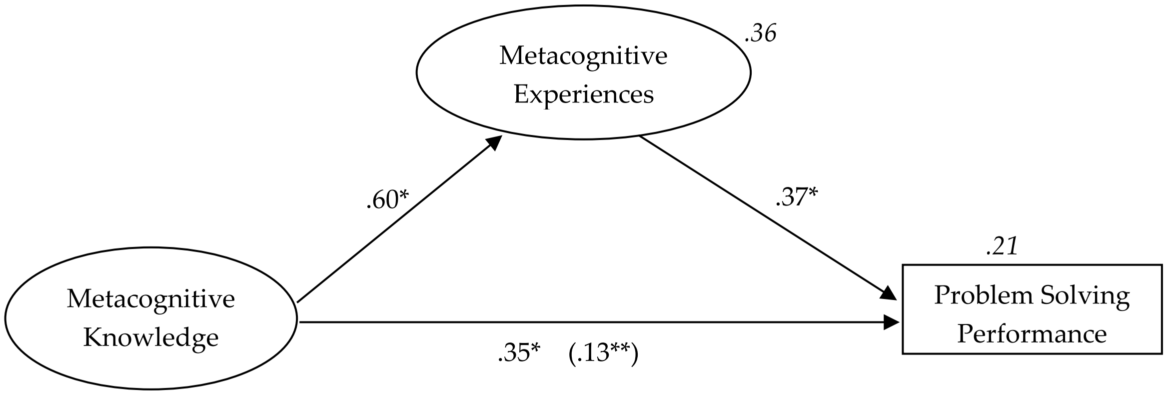

Figure 2. Standardized regression coefficients and variances for the relationship between metacognitive knowledge and problem solving performance as mediated by metacognitive experiences. The standardized regression coefficient between metacognitive knowledge and problem solving, controlling for metacognitive experiences, is in parentheses. * $p < .01$, ** $p < .05$.

If metacognitive experiences were controlled and the regression analysis was conducted without the mediator, the estimate of the causal path from metacognitive knowledge to problem solving performance would be significant (β= .35, $p < .01$). When metacognitive experiences were added to the model as a mediator, the direct effect of metacognitive knowledge on problem solving performance significantly decreased from .35 to .13. Thus, the model implies that metacognitive knowledge indirectly affects problem solving performance (β= .22, $p < .01$) through the mediation of metacognitive experiences. As illustrated in Figure 2, the mediation process implies that metacognitive knowledge had significant direct effects on both metacognitive experiences (β= .60, $p < .01$) and problem solving performance (β= .13, $p < .01$). This means that if metacognitive knowledge goes up 1 standard deviation, metacognitive experiences go up .60 standard deviations and problem solving performance .13. Besides, metacognitive experiences had a significant direct effect on problem solving performance (β= .37, $p < .01$).

A mediating variable can mediate a relationship in two forms: fully or partially. If the direct effect of the independent variable on the outcome variable is still significant while the mediator accounts for a significant amount of variance over the outcome, the mediating effect is interpreted as partially mediating the relationship between independent and dependent variables. On the other hand, if the direct effect becomes insignificant when mediating variable is added, the mediating variable fully mediates the relationship (Baron & Kenny, 1986; Shaver, 2005). The direct effect of metacognitive knowledge on problem solving performance (.13, $p<.05$) was low, but still significant at .05 level, in the hypothesized model. Therefore, potentially, metacognitive experiences partially –but close to fully- mediate the relationship between metacognitive knowledge and problem solving performance. The proposed model accounts for 21% of the variance in the problem solving performance.

The model showed a good model fit when the other two problems were also taken into account according to the chi square ratios (x^2/df) and various SEM fit statistics and indices are demonstrated in Table 4. The chi-square ratios and fit indices showed that the hypothesized model fit the data well for each problem.

Table 4. The Fit Indices of the Model Analysis for Each Problem

	Problem-1	**Problem-2**	**Problem-3**
x^2 / df (≤ 3)	2.12	1.58	1.53
GFI (>.90)	.97	.98	.98
AGFI (>.90)	.95	.96	.96
NNFI (>.90)	.97	.98	.98
CFI (>.95)	.98	.99	.99
RMSEA (<.08)	.05	.04	.03
SRMR (<.05)	.03	.02	.03

For all three problems, similar statistically significant standardized regression (β) and squared correlation coefficients (R^2) were obtained. These coefficients are presented on Table 5. Taking all three problems into account, it can be said that metacognitive experiences partially mediate the relationship between metacognitive knowledge and problem solving performance.

Table 5. Standardized Regression and Squared Correlation Coefficients for Each Problem

Standardized Regressions (β)	**Problem-1**	**Problem-2**	**Problem-3**
Metacognitive Knowledge → Problem Solving (Dirrect effect)	.13**	.19*	.16*
Metacognitive Knowledge → Problem Solving (Indirect Effect)	.22*	.19*	.17*
Metacognitive Knowledge → Problem Solving (Total Etki)	.35*	.38*	.33*
Metacognitive Knowledge → Metacognitive Experiences	.60*	.56*	.55*
Metacognitive Experiences → Problem Solving	.37*	.34*	.30*
Squared Correlation Coefficients (R^2)			
Problem Solving	.21	.23	.17
Metacognitive Experiences	.36	.32	.30

* $p < .01$, ** $p < .05$.

Discussion and Conclusion

This study was designed to test the role of metacognitive experiences in explaining the association between metacognitive knowledge and problem solving performance. The research questions dictated the use of SEM as it allows testing of theoretical propositions regarding the link between three constructs. Findings from the data analyses provided evidence for significant relationships between metacognitive knowledge, metacognitive experiences and problem solving performance. The data showed good fit with the proposed model. The results demonstrated that student's metacognitive knowledge was predictive of increases in problem solving performance, and that this association was almost completely mediated by metacognitive experiences. Given the importance of metacognitive experience as a mediating variable, it is worth mentioning that there is an indirect effect of student's metacognitive knowledge on problem solving performance.

The results revealed a low but statistically significant association between students' metacognitive knowledge and problem solving performance. The findings indicated that problem solving performance was significantly correlated with metacognitive knowledge in self-checking, evaluation, awareness and in the use of cognitive strategies. The finding on the direct effect of metacognitive knowledge is somewhat consistent with previous research findings (Hargrove & Nietfeld, 2015; Kitsansas, 2002; Sundre & Kitsansas, 2004). On the other hand, there are some studies that do not find a significant relationship between metacognition knowledge and achievement (Hong & Peng, 2008; Kuyper et al., 2000; Malpass et al., 1999; Purpura, 1997; Schraw, 1997). As discussed previously (Hong & Peng, 2008), the contradictory findings might be because of cultural diversity. To be more precise, it might be better to say that the findings indicated a significant relationship for Turkish students.

When metacognitive experiences were included in the analysis, the metacognitive mechanisms and relations became more obvious. The mediation model revealed that a great majority of the observed effect of metacognitive knowledge emerged as an indirect effect through the mediation of metacognitive experiences in task processing. Students perceiving themselves as having a high metacognitive regulation tend to have more consistent on-task feelings and estimates on a given task. It points out that students will be more conscious in their cognitive steps to solve the given problem. Metacognitive experiences, as conscious cognitive experiences, account for the association between metacognitive knowledge and problem solving performance to a large extent. This finding is mostly consistent with the result that emphasizes metacognitive experiences as "the missing link" (Efklides, 2006a, 2009), in our understanding of the metacognitive processes. Metacognitive knowledge functions at the macro-level, unlike metacognitive experiences, which function at the micro-level (Efklides, 2006a). Since metacognitive experiences are the area least focused by educators (Efklides, 2009), this finding illustrates the crucial role of what students' feel on a given task in learning (Aşık & Sevimli, 2015; Baars, van Gog, de Bruin, & Paas, 2017; Mihalca et al., 2017).

Measurement of metacognition is a highly challenging area (Schraw & Impara, 2000; Veenman et al., 2006). Self-report instruments are widely used in research settings but their usability is a recent concern in scientific discussions (Veenman, 2011a; Veenman, 2015). Many researchers suggest using a combination of measuring methods to obtain a deep insight into learning (Schellings & Van Hout-Wolters, 2011). One of the main goals in this study is to examine whether metacognitive regulation assessed through self-report questionnaires is related to the online measures during task performance. Data resulting from this study provided some evidence to understand the role of self-report instruments in assessing students' metacognitive skills. Despite the fact that students' metacognitive knowledge assessed by self-report instruments does not always seem to have an effect on performance, the results indicate that they may have an indirect effect of fostering problem solving performance through metacognitive experiences. Although some studies emphasized that declarative metacognitive knowledge does not guarantee actual use for the regulation of behavior (Veenman, 2013; Veenman et al., 2006; Winne, 1996), this finding illustrates how metacognitive knowledge, with metacognitive experiences, serves to promote problem solving.

Another important issue for researchers and educators in the study is to use a single problem to assess the role of metacognitive experiences in explaining problem solving performance. The data showed a good fit with the mediation model on a single problem. Similar fit indices were obtained for all three problems separately. Getting similar results indicates the fact that working with one problem can be sufficient to get satisfactory results in metacognitive experience and in problem solving studies (Winne, 2016). The identical results, obtained through the replication of problems, supports validation of single-item studies in assessing metacognitive facilities. This finding is consistent with the findings emphasized in the literature (Ainley & Patrick, 2006; Efklides, 2006a).

Teachers should have knowledge of teaching methods that support the development of metacognition and metacognitive teaching. In other words, teacher's pedagogical knowledge on metacognition is an important factor that enhances students' metacognitive development (Aşık, 2015, Jiang et al., 2016; Özcan & Erktin, 2015). Teachers who aim to support the development of students' metacognition need be able to consistently assess students' metacognitive processes. It is important that the teacher has knowledge and awareness on students' metacognitive skills in the classroom in order to support their development as self-regulated learners (Lee, Irving, Pape, & Owens, 2015). Therefore, using a single-item measure also makes the assessment of metacognition more accessible for educators.

Suggestions

The results indicated the importance of metacognitive experiences and metacognitive knowledge for problem solving performance. The importance of metacognition is gradually increasing in the research studies carried out in our country (Baş & Sığırlı, 2017) and it is also emphasized in the Ministry of National Education curriculum studies (Ministry of National Education, 2015). Further studies with different kinds of measurement instruments and different samples are expected to make contributions to the related study fields.

The teaching implications of the findings need to be studied. Classroom practices enhancing metacognitive experiences can be used in intervention studies to further investigate their effects on problem solving performance and achievement (Özcan & Erktin, 2015). The role of trait metacognitive knowledge in the possible "illusions of feelings" (Efklides, 2006b; Schwarz, 2010) between prospective and retrospective metacognitive experiences might be another research topic for the future.

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