Education and Science tedmem

Vol 43 (2018) No 193 121-141

Examination of Mathematical Values in Classroom Practices: A Case Study of Secondary Mathematics Teachers *

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Abstract

Teachers' preferences in decision-making by means of their pedagogical content knowledge about how students can learn better compose the values. These decisions reflect their personal values. Thereby, hints for the values owned by them are found in classroom practices when required in their decision-making. The purpose of the current study is to reveal the mathematical values of secondary mathematics teachers reflected in during the classroom practices. It was designed as a case study which is one of the qualitative research designs. Participants were 5 secondary mathematics teachers working at various secondary schools. Sampling was determined via the purposeful sampling method. Data collection tools consisted of semi-structured interviews based on the class scenarios and class observations which were video-recorded. Data was analyzed by content analysis method and according to the results, it was found that while teachers had values regarding the mathematics even if insufficiently in some dimensions, they didn't reflect other dimensions in classroom practices. They reflected objectivism, control and openness values more than complementary value duals. Finally, discussion and suggestions for mathematics education are presented in the light of the findings.

Keywords

Value
Categorization of values
Mathematical values
Decision-making
Mathematics teachers

Article Info

Received: 03.02.2017
Accepted: 01.25.2018
Online Published: 03.01.2018

DOI: 10.15390/EB.2018.7177

* This article is derived from Fatma Nur Aktaş's master's thesis entitled "Mathematical values and their reflections on classroom practices", conducted under the supervision of Ziya Argün. This article is also an extended version of the paper presented at the I. National Values Education Congress.

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Introduction

Values including the preferences between the possibilities and decision-making process play role in individuals' behaviors or decision-making consciously and unconsciously (FitzSimons, Bishop, Seah, & Clarkson, 2001). Teachers are influenced from the affective factors acquired in professional life experiences in decision-making how their students can learn better. Therefore, there are hints for the values based upon the ways of teaching in classroom practices (Bills & Husbands, 2005b; Bishop & Clarkson, 1998). In other words, decisions of the teachers through the available pedagogical knowledge represent teachers' personal values (Bishop, 2008a).

For effective teaching, a teacher needs to have subject and pedagogical knowledge, effective planning and management skills (Husband, 1947; Muijs & Reynolds, 2010, pp. 2-3). The teacher decides when and how to use this knowledge during the classroom practices. Decision-making skill is improved as they gain experience. In order to understand the reasons of the decisions, it is necessary to disclose values of the teacher underlying the decisions (Seah & Bishop, 2006). Research problem based on this rationale is "What are the values of mathematics teachers, who work at secondary schools and have occupational seniority difference, emerging in classroom practices?"

Value

Definition of value depends on the belief of the individual about what value is. In other words, its definition is individual (Southwell, 1995). This approach proposes values to be studied individually. Common ground of the definitions of values consist of making a selection, making a decision and demanding expressions (Sarı, 2005, p. 76). While Matthews (2001) define values as the means or pioneers of the behaviors, Halstead and Taylor (2000) define them as 'principles leading behaviors, good or desirable actions'. In the light of those definitions, from a broader perspective:

> Values are continuous principles in similar conditions leading the preferences and decisions taken in consideration of the significance, worth, demand of an object, idea, notion or behavior (Aktaş, 2014).

It is difficult to perceive similarities and differences for concepts of affective domain such as belief and value (Mcleod, 1992; Seah & Bishop, 2000). Kluckhohn (1962) emphasizes that one important point that sets apart values from beliefs is '*engagement commitment in situations involving possible alternatives*' (p. 559). Based on this oppinion, Clarkson and Bishop (1999) evaluate values as beliefs in action. Another important point is that while beliefs can be divided into categories such as wrong and right, Kluckhohn states that values are concerned with matters that are important or not important in reality (1962; as cited in Seah, 2002; Kluckhohn, 1973). The right or wrong judgment of a fact is probable or meaningful on the basis of some facts. In other words, while belief is expressed regarding the content, value is expressed independent of the content of what the value is. '*Mathematics is enjoyable*' belief, in particular, reflects a correct or false judgment of mathematics. On the other hand, value does not necessitates a special case for the desired or undesired idea. An individual adopting the '*enjoyable*' value will highlight this value in different situations (Bishop, Seah, & Chin, 2003, pp. 725-727). From the point of this research study, values in mathematics are indicators of how well mathematics teachers can apply their beliefs about classroom practices (Clarkson, Bishop, FitzSimons, & Seah, 2000).

Something that is considered important by the individual (related to the values of the individual) reflects to what is considered to be right (related to the beliefs of the individual) developed by the individual within the social context in time and accepted as true. Therefore, values internalized by the individual are important. From the point of the current study, (Bishop, Clarkson, FitzSimons, & Seah, 2000; Seah, 2008), teachers who utilize technology as an important tool in teaching and learning mathematics are expected to have the belief that the use interactive whiteboard in the classroom increase the classroom participation of students. To give another example, we can say that the belief 'even if the numerical value is wrong, the student who applies the correct method deserves a full grade 'is a sign that a teacher has objectivity value (Seah & Bishop, 2002).

Value and Decision-Making

Values effect the preferences of probable actions of the individuals. However, individual values don't portray the nature of people's decisions, actions and their results. In the process of making a preference and decision-making among the alternatives, value system plays role rather than individual values (Bishop et al., 2003, pp. 721-725). Decision-making is the sum of physical and cognitive efforts about selection and preference of various conditions (Taşçı, 2011). When we examine the studies on the factors affecting the decision-making process, variables vary (Bishop & Whitfield 1972; Jacobs, Lamb, & Philipp, 2010; Schoenfeld, 2011; Shavelson & Stern, 1981). The common point of these studies is the emphasis on the beliefs and / or values playing rol in the decision-making process. In the studies that investigate the instant decision making process in classroom applications, Schoenfeld (2011) dealt with beliefs, while Bishop (2008e, pp. 29-35) studied the values. Since, goals and conditions vary under different conditions during decision-making, preferred values in actions also change. For a teacher, in possible similar conditions and contents, prior values remain unchanged. These values are privileged to another one at another decision-making point. Hereby, teacher's value system is the basis (Bishop et al., 2003, pp. 722-729). Bishop and Whitfield (1972) indicated that teachers interpret their decision-making in daily teaching conditions through life experiences in general, however, more peculiarly, they interpret this process via past knowledge through the educational experience and goals about the courses (as cited in Borko, Roberts, & Shavelson, 2008).

Categorization of Values and Mathematical Values

'Values' notion is not new in education like in anthropology and organizational studies (Harmin & Simon, 1967). But, relatively, 'values' is a new research field in mathematics education (Seah, Bishop, FitzSimons, & Clarkson, 2001). Bishop's (1988) research about culture and studies on ethnomathematics made values focus.

Sam and Ernest (1997) categorized values explicitly revealed or implicitly developed in the process of planning, teaching and learning of mathematics education in consideration to Bishop's studies. *(i) Epistemological Values:* They are the values found in the epistemology of the teaching and learning processes. *(ii) Social and Cultural Values:* They are the values in relation to human's responsibilities for society in mathematics education. *(iii) Personal Values:* They are the values that affect person as an individual or a learner. While Durmuş, Bıçak, and Çakır (2008) categorized the mathematical values of science and technology, mathematics and primary school teachers, they took Clarkson et al.'s (2000) categorization as a basis. They categorized those values as the '*objectivist*' values relying on behaviorist and cognitive approaches and '*subjectivist*' values relying on constructivist approach. Dede (2013a) examined the values found in decision-making processes of Turkish and German mathematics teachers as *productivity*, *socialization, authority-flexibility and gender differences* in the Values in Mathematics Teaching in Turkey and Germany project. In this study, continuous comparative analysis with the data obtained as the result of the semi-structured interviews made with the participants, categories *productivity, socialization, authority-flexibility and gender differences* were obtained.

White (1959), having defined the cultural growth concept which he thought its being transformed through the technology, defined its *ideological, sentimental* and *sociological* dimensions. Meanwhile, Bishop (1991a, pp. 62-81) claimed that math is one of those technologies and defined values of mathematical culture based on those three dimensions. *Ideological dimension*; values of individuals learning and teaching mathematics, *individual dimension*; personal values of the individual in the context of teaching and learning, *social dimension* is about the value preferences of the individual about math and society (Bishop, 2008d). Bishop, FitzSimons, Seah, & Clarks (1999) categorized the values taught in mathematics as *educational values, mathematical values* and *mathematics educational values*.

Mathematical values are the values reflecting the scientific nature of mathematical knowledge produced by the contributions of diverse cultural backgrounded mathematicians (Bishop et al., 1999). Bishop (2004) categorized the values gained by teachers and observed in math classes into three complementary groups. Ideological duals; rationalism and objectivism, sentimental values leading the mathematical development; control and progress, and sociological values indicating the relationships among the people; openness and mystery (Bishop, 1988, p. 82; Bishop et al., 1999). This set consists of the values taking attention and possibly to be emerged in diverse cultures (Bishop et al., 2003, p. 205).

Ideology: Rationalism-Objectivism. Rationalism claiming the inductive method to reach the only right way denies the heuristic pragmatism and practical rules. When analyzing a reasoning and disproving a hypothesis, rationalism value guides us. Therefore, proving, abstraction and theorization are the reflections of rationalism in decision making process (Bishop, 1991a, pp. 61-65, 1991b, pp. 201-202; Bishop et al., 2003, p. 720). While rationalism is about the rationale of the relationship between the ideas, its complementary dual, objectivism, is about the generation of ideas and facts. Accordingly, features and relations effecting decision making are called as objects. This denomination enables to deal with the Symbols in mathematical knowledge and abstract assets as if they are objects (Bishop, 1991b, p. 202; Bishop et al., 2003).

Sentiment: Control – Progress. Mathematics has always controllable correct answers (Bishop et al., 1999). This process occurs via algorithms, rules and criteria. Discipline to mathematical rules is learnt through those facts (Bishop, 199a, pp. 69-72). Control value, while finding the only one mathematical solution method sufficient, makes a generalization this method to other problems. However, inverse sampling and exceptions lead to another new problem (Bishop, 1991b, p. 203). Thus, mathematical abstraction is a matter enabling generalization of a problem from an already known problem. In the meantime, the individuals who are aware of this process reflect progress value through the evaluation of alternatives like the definitions existing in the nature of mathematics, algorithms and proofs (Bishop, 1991a, pp. 72-75).

Sociology: Openness- Mystery. Mathematical knowledge is the fact to be validated regardless of time, place and people (Bishop, 1991a, 1991b, pp. 203-204). Teachers' wish to explain the mathematical facts from the students rather than accepting them as truths is because of the openness idea. Thus, it provides the person a democratic way to express his ideas (Bishop, 1991a, pp. 75-77). Although mathematical culture has openness value, perception of mathematics as a mysterious phenomenon is a paradox (Bishop, 1991a, pp. 77-81). This stems from the abstraction regardless of content and surprises in its nature (Bishop, 1991b, p. 2204; Bishop et al., 2000; Dede, 2007).

Mathematical Values and Classroom Practices

Class is the complicated structure under the social and cultural influence. Remarkable variables effecting the classroom practices are mathematics as a discipline, mathematics as an educational field and values of teachers (Bishop et al., 1999; Handal, 2003). Bishop (1991b, p. 199) defined the educational practices emerging the values at four levels: Societal level, institutional level, pedagogical level and individual level. *Societal level*; refers to the values based on mathematical knowledge, found in official curricula and among the entrance conditions to higher educational programs. *Institutional level* comprises of the role of mathematics in school curricula and classroom practices. *Pedagogical level* is about the emphasis of a single quality of mathematics by teacher. *Individual level* refers to the personal importance attributed by student to be successful in mathematics. Bishop (2008a), stated that values of teachers depend on their pedagogical identity. In mathematics teaching process, classroom practices such as discussions and diverse problem solving methods, small group works are significant practices reflecting the rationalism and openness values (Bishop, 1991b, pp. 207-212).

The Purpose and Significance of the Study

The aim of this study was to define the mathematical values of secondary school mathematicss teachers influencing the teaching processes in math classes and examine their reflections on their preferences and decision making processes.

Values play an important role in the selection of the problem solution, solution method, emergence of the concepts in solution process and evaluation (Ernest, 2008). In consideration of those variables, decision of the course targets, arranging the learning experiences and evaluation are values expected from teacher (Durmuş & Bıçak, 2006; Ernest, 1989). Therefore, it is necessary to focus on the values of teachers for effective mathematics education (Bishop & Clarkson, 1998). According to Bishop et al. (1999), More researches about the values gained and taught by teachers and impact of values in educational practices are needed this situation sets an obstacle on equal emphasis on math in the classroom environment, which provides an effective mathematics learning and teaching environment. Because of this, it is necessary to conduct basic research on values for designing effective learning environments. When studies based on this need are examined, researches on the reflection of decision process and individual values are important. In studies conducted in Turkey, values found in textbooks were examined, paid significance to values education, and values of teachers set forth through limited values like positivist and constructivist perspectives (Dede, 2006a, 2006b; Demir, Somuncu Demir, & Durmuş, 2012; Durmuş et al., 2008). Dede (2013a, 2013b, 2014) studied the values of German and Turkish mathematics teachers in terms of the various variables and impacts of those values. While, studies in literature studies focused on the analysis of values gained by the mathematics teachers, in practice, studies on how these values emerged were relatively limited (Bills & Husbands, 2005a, 2005b). Thus, teachers' in-class practices, decisions and behaviors and reasons for their choice will be more understandable. Accordingly, the in-service training that will be design and maintain the values education that is among the objectives of the teaching program to work in this direction is likely to include the classroom practices which will enable to reflect them the values effective in decision making such as planning, choosing textbooks or materials suitable for teaching purposes, evaluating and assignment, and group activities etc. In addition, it is considered that the study will help the experts on the development and modification of the education and teaching of values and values in the secondary school mathematics curriculum.

Method

Research Design

In the research of values in mathematics education, one of the ways to make the measurements objective, valid and reliable is qualitative research design (Clarkson et al., 2000; Seah, 2008). Case study is a research method examining the situations in a real life frame and enabling the presentation of the study from a holistic perspective when there is more than one data source (Merriam, 1998, p. 19). Values that a teacher has are reflected in class practices openly or secretly (Bishop & Clarkson, 1998). For this reason, interviewing and observing in the determination of open and confidential values allow to make comments and deductions in a casual situation. Considering the purpose of the research, the research is designed in a case study design which is one type of qualitative research designs in order to examine the values deeply in the current class environment and to determine their reflection. Merriam (2013, p. 40) deals with the case study as a field study examining a phenomenon in real life context. Similarly, in this study, participants are examined regarding their classroom practices and contextual views. Each teacher who participated in the survey which is in a collective case study pattern generates the situations of the study. Since each situation is different within the study, the study is designed in a multi-state pattern.

Participants

One of the variables gained by the mathematics teachers and reflecting variable in their practices is school type (Bills & Husbands, 2005b; Bishop, 2008c, pp. 191-203). So that, researches started with 8 secondary mathematics teachers working at various secondary schools, 5 participants from whom necessary and satisfying data were gathered were included in the study. Participants were working at various secondary schools in Ankara and occupational seniority levels differed from each other. Another variable effecting the mathematical values is content (Bills & Husbands, 2005a; Bishop, 2008c, pp. 191-203). The research was conducted with mathematics teachers teaching 9th graders whose program included diverse contents by taking education program into consideration. Since the study was maintained with a researcher, preferences of the teachers were paid attention and observable classes were selected. In addition, regarding the pre-service training, occupational succession, and experience that affect the teacher competencies expected from an effective teacher were diversified. On the voluntary basis, participants were selected through maximum variation and stratified sampling methods Table 1 refers to the information of the participants named as T1, T2, T3, T4 and T5. Availability of 5 cases (teachers) having different occupational seniority levels in this study is a limitation in discussion of the results. Therefore, description of the participants and evaluation of the results from this perspective will contribute to the discussion.

Table 1. Participant Information

Teacher	Occupational Seniority Level (Year)	Graduated Department	Past Experiences
T1	2	Department of Mathematics & Non-Thesis Master Program	Vocational Secondary School
T2	7	Department of Mathematics & Non-Thesis Master Program Lisans	Prep School
T3	15	Department of Mathematics & Pedagogical Formation Education	Prep School / Primary School / General Secondary School
T4	20	Department of Mathematics Education	Vocational Secondary School / General Secondary School
T5	26	Department of Mathematics Education	Middle School / Multi Program Secondary School / Anatolian Secondary School / General Secondary School

Data Collection Instruments and Procedure

Data collection instruments of this study were semi-structured interview forms, video recordings of the classes and unstructured observation forms. Prior data sources were interview questions based on class scenarios. It was found that participants were influenced by the interview questions in the conducted pilot study. For reliability, observations were decided to conduct before interviews. Values reflected from teachers may stem from the values of education program, contents or course textbook (Atweh & Seah, 2007; Bills & Husbands, 2005b; Bishop, 1997). Therefore, in preparation of the interview questions, class levels enabling observations of diverse contents and learning domains were determined. Semi-structured interviews during and after the course observations and "numbers" as sub learning content of 9th grade Algebra Learning Domains in Mathematics Course Curricula at Secondary Schools were taken. For reliability and reach rich data, course observations of sublearning contents were implemented

Preparation of Semi-structured Interview Forms. By taking into content impact account on the reflected mathematical values in classroom practices (FitzSimons et al., 2001), interview questions were prepared for the outcomes of sub-learning domains such as integers, rational numbers, modular arithmetic and real numbers. Classroom scenarios were designed based on activity hints, explanations and previous studies in literature for interview questions in the syllabus (Bills & Husbands, 2005a; Bishop, 2008b; Bishop et al., 2000; Bishop, Clarke, Corrigan, & Gunstone, 2005; Seah et al., 2001; Seah, 2004; Seah & Bishop, 2000; Dede, 2006a).

Interview questions were placed in classroom scenarios which are descriptions of 4 class processes including learning outcomes for the mentioned contents for an imaginary teacher. It was aimed to have teachers share their own values and thoughts by feeling as if experiencing the stated situation through those scenarios. Since, the decisions of teachers would reflect their values (Bishop et al., 2000), classroom scenarios and interview questions were designed to have them make decisions. Interview form including 43 questions were made up by taking expert opinion from academicians who had studies on values in mathematics education and by conducting a pilot study with a research assistant at Department of Mathematics Education and 2 secondary mathematics teachers. In approximately 90-95 minute lasting interviews, prompts were given by taking benefit of video-recording in classes and via teacher opinions in order to increase the validation. While they sometimes helped in understanding the partcipants' expressions better, they sometimes enabled finding out the values of the participants thorugh the sample cases by utilizing classroom observations. Interviews were conducted at secondary schools when the silence was kept. Therefore, long-lasting interactions with the participants were held during the course video processes, participants are assumed of giving sincere answers. Besides interviews were saved via video and audio records.

A part of scenarios in the interviews and its questions are below:

"(…) Pelin teacher indicated the set of composite numbers ($\mathbb{Z}$/m) according to the division rule by whole numbers and planned a task in order to show the features of modular arithmetic operations. Firstly, she asked students to find composite numbers from the division of 17 and 14 to 3. Next, she asked them to calculate the composite numbers from the division by 3 from the addition and multiplication of the obtained number. And for a while later, she said: "Let's find the composite numbers of the division by 3 from the addition and multiplication of 17 and 14. Compare the results you found and please, share your ideas with class".

- a ≡ x1 (mod 3), b ≡ x2 (mod 3)

  In order to calculate the equal according to 3 module of a+b and a.b, what are your thoughts about the necessity of their discussion? Please, explain it.

- Please, share your approach you used in your classes for this learning outcome in this operation."

Class Observations. According to Seah and Bishop (2000), an algorithm accepted as the best to teach content by a textbook or teacher can be used. Accordingly, saying that teacher had only control value is wrong. Therefore, courses ranging from 6 to 9 course hours were video-recorded in order to examine values obtainable and various contents in mathematics education in a general dimension instead of determining teachers' values for a single content and observation for classroom practices. Before the courses to be video-recorded, for long interactions and natural atmosphere (Bishop, 2008c; pp. 191-203; Glesne, 2011), sample video recordings were done. After daily check, teachers' advises were demanded when required, so that participant confirmations fulfilled. During video-recordings, natural class atmosphere was kept and they were made from a point of view enabling to cover two thirds of the whole class. Additionally, by using tripod, situations out of the vantage point were also recorded. Furthermore, unstructured observation form was used for the points thought remarkable by the researcher and missed during the observations. The role of the investigator in course observations is the participant observer . Some changes in the weekly lesson program during the observations prevented the recording of some courses. This is a limitation of the study.

Data Analysis

Case study as a qualitative research design includes the depth examination of by analyzing the data collected through observations and interviews (Glesne, 2011). Thus, mathematical values as the unit of analysis of the current study were examined via content analysis in the light of data collected by interview and observations.

According to Merriam (1998), coding list, which is the first stage of coding, was made up via the previous studies. Additionally, video-recorded courses were worked out by the researcher. Firstly, data obtained the research were just read by keeping in mind the aim and the problem of the study. Next, raw data were coded for each value. While coding, key words in the interviews were underlined and highlighted notes were taken. Value categories were constructed via obtained data from coding and literature reviews (Cao, Seah, & Bishop, 2006). In coding made up based on the literature reviews, some differences occurred as a result of first readings and coding. For example, '*effort for the exploration of cause and effect relation and explanation*' code was added. Some of the codes obtained for the value themes are presented in Table 2.

Values gained by the teacher emerged seemingly in decision-making stage (Clarkson et al., 2000). So that, points of decision-making of teachers in video analysis were taken particularly into account. In order to detect the consistency of the participants' answers, analysis were made out of the similar categories and based on the data obtained from course observations and interviews. The following is an example of the coding provided in the observation in order to support the interview data. However, some of the data which could not be obtained by interview were reached on the course observations. For example; the following decision-making situation was recorded in the course observation of the participant from whom no data was obtained over the placement of the proving in their interviews:

> *T3: Is not it difficult to write a set of irrational numbers? Which is irrational, which is rational? For example, what is the guarantee that $\sqrt{2}$ is irrational? (waits for a few seconds, "$\sqrt{2}$ is a rational number?" and writes the question to the throne), is our question actually saying $\sqrt{2}$ is irrational? Do you remember that at the end of the first period, we learnt the contradiction proving?*
> *Students: Yes.*
> *T3: We will solve this by proving for not being in the proof. […] (T3 solves the problem)*

The wish to present proof that $\sqrt{2}$ is irrational here is a critical condition for the teacher to decide. Additionally, it is a reflection of rationalism value in classroom practices.

Table 2. Coding Examples of Mathematical Values (Aktaş, 2014)

Values	Codes	Description
Rationalism	Proof	Teaching proof and making effort for mathematical proof
	Discussion	Work for inquiry, ratiocination and discussion
Objectivism	Setting a symbol, model	Endeavoring to set symbols, models, diagrams etc and encouraging to use them
	Concretizing	Struggling for concretizing of the mathematical knowledge
Control	Practice of targeted outcome	Working out for the development of outcome based operations and strengthening of practical skills
	Associating with daily life	Making effort to indicate the math as applicable in daily problems and sampling about this.
Progress	Development of mathematical knowledge	Struggling for the activities related to development of mathematical knowledge and operating them
	Generalizing	Making effort to have the students reach generalizations
Openness	Freedom of expression	Targeting to have the students improve expressive skills in clear and open way.
	Justifying	Working out to teach justifying method and techniques
Mystery	Raising awareness and admiration	Seeking for the ways to raise awareness and admiration through significant ideas
	Have them surprise experiences	Having the students gain surprise experience through unexpected findings

There is analysis sampling (coding) obtained from interview data below. Next, one part of observation data which supported this data were giiven as an example. While first excerpt from the T2 interview set the example for generalization coding of progress value; second excerpt reflected openness value; having surprise experiences code of mystery value emerged in the last excerpt.

"…Student solves the question. But, this may work fort hat question, however, it doesnt work for other questions or numbers when we change. Yet, student states that it is ok. Just to Show that, by giving other numbers, repeat then. Let's see whether it fits or not?..."

"... Maybe there is an answer from root 5, but is it an exact number? Then, I will guide him a little more. Do all of the numbers in Z / 5 have the roots? It can be asked again whether it were in itself, like the roots, let's think... "

"I think it's convenient, because if you make student surprised by saying: "Was this like this? May be? "By creating contradictions, you lead them to considerable thought. And I think he's comprehending in this way. "

The following observation notes support the above mentioned interview data. This data is a sampling for all three value codes. While struggling to point out the divisibility rules by 2 via generalization value, T2 creates opportunities for help Merve confirm her idea. In this process, T2 uses surprise experiences leading the students to contradiction.

T2: Now that, divisibility rule by 8…

Merve: Ok. Take out the rule out of 4 and 2.

T2: Just a second, can 60 be divided to 4?

Merve: Yes.

T2: Can it be dividied by 2?

Merve: yes.

T2: 2 rule is ok with this but, it can't be divided by 8 […]

When this observation data is analyzed, it can be said that T2 reflects progress, openness and mystery values in the same critical situation in classroom application. This indicates that the individual reflects one or more values in the value system in classroom situations.

Trustworthiness

Cogency is about the setting of the research, participants in this setting and presentation of the situation by the events (Yıldırım & Şimşek, 2011). Naturality and conciseness of the interviews were provided by the course observations as a result of pilot studies and, therefore, through the long interactions. Course observations lasted about 5 weeks and it led the researcher longitudinal observations. Additionally, before the courses which were related the targeted gains, two courses were video-recorded. In this way, accommodation of the participants and students to the existence of the researcher in the setting and natural context were aimed. Participant confirmation was attained for the interesting points during the observations through prompts during interviews and after observations. Diversity in data sources via semi-structured interviews in the research and notes taken by the researcher. Furthermore, expert opinions were taken from an academician who had studies on values in mathematics education for the strengthening of analysis of collected data and obtained theme and categories. In this examination, 20% of the data gathered were coded. Among the codes obtained, 86,4% convergencewas gained. Differences in coding were discussed between the researcher and second coder again and met consensus. Accordingly, editions as mentioned above were done in coding.

Results

Findings were evaluated under 6 categories; rationalism, objectivism, control, progress, openness and mystery as the mathematical values gained by the teacher according to Bishop (1991a). Nicknames were used for the students in the classroom observations. Seperate disclosure of the findings for each participant prevented us from having a general perspective in terms of values of mathematics. For this reason, values of mathematics are categorised. Although the values were presented categorically, like in the conceptual framework, the individual's value system is wholly effective in the instant decision-making process. In other words, we can observe the reflection of more than one teacher value at the same time in such a critical situation. When analyzing the findings, it is useful to take into consideration from this perspective.

Rationalism

T1, T3 and T5 stated that on condition of the discussions or ratiocination and inquiries at unsatisfying classes at the readiness level, students would seesaw in mind. T5 expressing that she could allocate short time for reasoning and discussion or ratiocination at the explanation and progress stage, said that she didn't need discussions depending on the course content. In contrast to other participants, T2 preferred to direct the discussion by asking various questions and through hypothesis on these situations.

T2 and T5 think that it is important to give students opportunity to defend themselves and query for the examination of their cognitive processes depending on students' feedback. Emphasizing the significance of question and answer teaching method in this process, T2 said that query of students by means of several hints and feedback led them recognize the cause and effect relationship.

T2: If 3a5 three digit number is fully divisible by 3, what are the values a can get? Fulya… (raised her hand)

Fulya: It can get 1.

T2: Why?

Fulya: Because, when we put there 1, the sum is 9. And it can be fully divided to 3.

T2: All right, that is 8+a. Then, what else can it get?

Fulya: 4. Then, the sum is 12. And later, it makes 7.

T2: Any other?

Fulya: There is no other…

T2: Why not 10? […]

In the situation above, there is support for data obtained from interview. It is seen in the situation above that T2 interrogated Fulya' s answer and gave opportunity Fulya to defend herself. We can say that T2 reflected rationalism value to confirm Fulya' s answer in classroom application. T2, thinking that students should be grouped into two to progress productively, emphasized that she found difficult to allocate time for similar activities because of the intensive syllabus.

T3 said that reasoning and discussion or ratiocination were possible by using the question and answer method based on the sample situation or problems. Briefly, we can express he included rationalism value with only control value. In this process, T3 also mentioned the significance of content variable. T3If the content includes more abstract concepts for the students, T3 reflects rationalism value with the sub-dimension for concretizing, which is the result of objectivity value.

Participants thought that level of readiness of 9th graders was unsatisfying for demonstration and proving. Reflections of different value dimensions were identified, while participants' applications didn't reveal the reflection of the related sub-dimension of rationalism value.T2 adopted openness value by testing the accuracy of the mathematical rules or expressions at the explanation and deepening stage. While in the interviews with T3, there was no finding for the encouragement of students for the proving and demonstration, in the course observations, placing the proof of √2 as an irrational number in classes was seen. In addition to participant opinions, T4 and T5 thought that proving or reasoning didn't take place among the teaching program goals. However, T5, by considering that proving increases persistence of students' learning, stated he explained the demonstration of some mathematical expressions.

Objectivism

T1, T3, T4 and T5 emphasized that readiness level of the students is insufficient to write mathematical demonstration and make use of them. Because of this, they thought symbols should be explained according to control value. This is an indication of how participants reflect their control value in their practice rather than objectivity value of the participants in order to demonstrate their mathematical language skills. T4 stated that it is important to create conditions for students to be able to express themselves according to openness value so as to reach intended mathematical demonstration or model.

We can say that T2 gives place practices to objectivism value at the explanation stage frequently. She emphasized the importance of content to encourage the students to make diagrams or symbols and time allocated. We can say that T2 works for the use of mathematical symbols in the classroom practice below.

(explains the congruence class in modular arithmetic)

T2: Instead of writing the numbers whose composite number 1 from the division by 5 one by one, when we just score it with a line, what does it tell us?

Students: Repeating number…

T2: Repeating numbers are totally different, later we will talk about them.

Students: But, we were scoring a line on them, too?

T2: It was put on the repeating numbers after comma. But, this is not the same case, this is about the equality of 0… Putting a line since it represents all the numbers giving 0 composite number, when divided to 5 […]

T1, T4 and T5 think that objects representing the geometric shapes around us and area of use in daily life in order to concretize mathematical values expressions or concepts are significant. Additionally, T4, in order to have the student express their opinions, tried to concretize by the use of numerical axis, graphic and model. Thereby, T4 paid attentions to objectivism to reflect rationalism and progress values.

Control

We can say that T1 gave feedback about the accuracy of students' solutions at the evaluation stage to maintain the fluency of the operations and improve their operational skills. For this, he finds weekly assignments appropriate to detect the errors or misconception. T1 and T2 think assignments crucial with multiple choice questions for each learning outcome and operational skill development. At the end of each class, T2 delivered activities including 10-15 multiple choice questions and checks students' homework in order, and then, explained necessary points.

T3 and T5 think that problem solving method is more efficient in order to improve the operational skills of the students and after the presentation of the content and evaluate the acquisition of target outcome instead of discussing with students. Based on those findings obtained from interviews by being supported with observations, it can be said that in the observations of participants' lessons, control value in comparison to rationality value is more frequently confronted. Rationalism value in course observations was reflected frequently, as well. Despite the other participants, T4, in order to improve the operational skills stated that there was no need for too many examples. T4, gives place to an advanced problem case with "question of the day" title at the evaluation stage. In this stage, we can say that he pays attention to operational skills one by one and practice.

T1 and T2 think that association of the mathematical content with daily life takes students attention, however, it is not possible to associate each content in instructional program with daily life. It can be said that T2 tries to design problem situations in accordance with its mystery value or situations raising admiration and interesting ideas by relating them to daily life experiences. T1, T4 and T5 states that mathematics improves creative thinking skills, even if mathematics presents solutions for daily problems, but concrete examples cannot be found for each content.

T4: […] In daily life, everything may not be of your service. Let's give such an example, […] Think that you are watching the training of a football team. Goalkeeper is also running with the other players […] However, goalkeeper doesn't run during the 90 minutes of a match, just stands at the goal […] Namely, there is no must to use everything you learnt in daily life. When you pay interest to mathematics, your vantage point will improve, you are learning to think more logically.

T3 stated that he didn't arrange the mathematical situations associated with daily life. He used associations while teaching a lesson if it is confronted in course content according to the problem conditions. Like in the classroom practice below, T3, calls attention to the course contents and how they are applicable to daily life.

T3: […] the use of modular arithmetic is very broad. For example, let's say… astronomers… say a comet… for instance, Abel comet approaches to world once in 300 years or seem from the World. When was it seen? Let's say in 1872. When did it appear four times before? (writes the problem and solves) […]

Progress

T1, T2 and T5 gave some explanations for how mathematics developed in order to validate the mathematical expressions, point out the relationships among the concepts and reach generalizations. Thus, it can be said that participants reflected control value together with progress value.

T2 and T4 let students inquire the relationship among the concepts and use question and answer method for reasoning. They design problem solving activities to set a relationship with pre-learnings and target outcomes. The emphasis on the relationship between divisibility rules and equivalence notion at the explanation stage of the classroom practice of the modular arithmetic content was observed. In practice below, T3 made effort to make students understand the relationship between fractional number and rational number.

(He scans two pieces out of 4 equal pieces and 4 out of 8 equal pieces)

T3: Doesn't this indicate us ½ ? What happened? Did 2/4 and 4/8 pointed out the same size? Half. Let's open this a bit more, 50/100. They all indicate the same size amount. Then, I can use this expression for those fractional numbers. 2/4, 4/8, 50/100 and ½ were all equivalent fractions […] We will call ½ the representative of all these fractions […]

T4 and T5 said that it was necessary to intimidate the students the accuracy of the attained expression for generalization in every condition. For this, T4 finds problem solving necessary for the conditions providing and non-providing generalization of sample conditions.

Openness

T1, T2 and T5 think that explanation and practice are necessary on different sample conditions to have students observe that mathematical knowledge is reliable and transparent for a designed scenario found also in the interview " to obtain equivalence according to 3 module, on condition that $a \equiv x_1 \pmod 3$, $b \equiv x_2 \pmod 3$, a+b and a.b" . It can be concluded that T1, T2, T4 and T5 placed sample situations to justify that ideas claimed or reached generalizations, formulas set forth are valid for all conditions.

As in the classroom practice below, we can say that, T2 pays importance to openness value to show ta accuracy of the knowledge. In addition, T2 gave opportunity to Kadir defend his answer and let him observe his mistake by creating dilemma.

T2: What is the composite number from the division of 6161616… 61 by 8?

Student A: 0

Student B: Can 61 be divided to 8?

Kadir: 1

T2: How did you calculate? (looks at the student's notebook)

Merve: We will divide 161 to 8.

(call Kadir to board)

T2: Come and explain how you did.

Kadir: I divided 616 to 8.

T2: Why?

Kadir: This was at the end.

T2: If I said 60-digit numbers, it would be wrong […]

T2 used question-answer method to have students express their own opinions at the explanation stage. At the evaluation step, he gave opportunity to explain the given answers and defend. T1 and T4, instead of discussion or ratiocination, enabled students share individually regarding the content. T4 and T5 consider the mathematical symbols and expressions or generalizations correct in all conditions instead of proof and demonstration methods and trying the mathematical knowledge intimidate clear and reliable. Therefore, on the conditions when rationalism value reflection is expected, they adopt openness value.

T3 said that he has the students prepare boards to share admirable mathematical operations and ideas and arrange mathematics club activities. T5 was observed that he gave opportunity to share students' their opinions regarding the association between integers and their use in daily life. Thus, we can conclude that participants reflected control and openness values together on decision making in classroom applications.

Mystery

We can say that participants reflected mystery value with openness, control and progress in classroom practices. T2T2 thinks that questions bear conflict in the students' minds and raised curiosity, and this curiosity led them inquire. T2T2 and T5, similar to the difference in occupational seniorityasked generally questions including conflicts through the problem situations. Classroom practice given below shows how T2 tried to raise admiration and curiosity to explain the ways to reach the divisibility rules.

T2: […] can be divided fully to 3[…] Well, why do we sum all the numbers? Have you ever thought? [...]Did the mathematicians come together and say let's decide the divisibility rule like this? Did they find somewhere? Didn't they?

(After a short discussion, he explains.)

Student: Oh… I got it!

T2: […] Almost all the rules come out from this solution. (Numerical analysis) […]

We can say that in T2's classroom practice, mystery dimension was observed together with rationalism and openness at the stages of discovery and explanation. In order to lead Merve verify her own idea for the accuracy, he designed a sample situation to have her surprise experience, and directed her. In this course observation, control value and mystery values reflections were observed.

T2: Ok, now, divisibility rule to 8.

Merve: If the divisibility rule to 4 and 2, then, it can be divided to 8.

T2: Just a second. Can 60 be divided to 4?

Merve: Yes.

T2: To 2?

Merve: Yes.

T2: It fulfills both of the rules but can't be divided to 8 […]

T3 stated that he selected sample situations to raise curiosity and admiration according to suitability of the time being instantly. Since he cannot predict the level of readiness level, he thinks that the examples won't take students' attention or students won't reach to the solutions. Furthermore, he prepares boards where students can express themselves and share interesting mathematical problems or knowledge. Shortly, T3 thinks the mystery dimension together with the openness value for the sake of the students' join.

T3, T4 and T5 arrange sample situations about how mathematical content find solutions of daily life problems to raise admiration and curiosity. Additionally, that T3 gives places mathematical stores or explanations was also observed.

(He defines the irrational number notion and gives examples. He proves that $\sqrt{2}$ is an irrational number.)

T3: People didn't deal with $\sqrt{3}$ number as much as they did for π […] they are obsessed about π. It is like a magic number.

Kaan: Let's just write 3.

T3: You cannot just write 3 and go for the next. Like you, in medieval ages, since 3 was accepted holy, there were people taking π as 3. (He writes the decimal part of π in 20 digits. This part continues fully 200 digits more, then, changes […]

Discussion and Suggestions

In this part of the study, discussion on the basis of Bishop (1991a, 1991b) and Bishop and his colleagues' (1999) value categorizations was held and suggestions were presented for the mathematics education.

According to the results, teachers didn't reflect the mathematical value duals equally like in the study by Seah and Bishop (2000). There was no participant reflecting at least one dimension of all during the whole class in classroom practices. However, values gained by the teachers in classroom practices are crucially important in practices such as setting the course objectives which emerge in instructional stages, managing the learning experiences for those goals and control to what extent the goals reached, corrections of the deficiency or errors etc (Ernest, 1989). Therefore, we can conclude that teacher should create diversity and develop themselves to reflect the values more. Even in the curriculum supporting values education (Ministry of National Education [MNE], 2017), only moral values are included. It is therefore possible to provide opportunities for teachers to improve themselves regarding the values emerging from the nature of mathematics. The results are recommendations so as to inform our teachers and experts about the value of mathematical values and provide a place for our teaching program and for the experts and related institutions developing programs.

Query, discussion or ratiocination dimensions based on the induction which constructs the basis of rationalism are significant (Bishop, 1991a, p. 62). Since discussions are not held in math classes in general, teachers require supportive materials and different strategies. This situation emerges one of the most important areas that teachers should improve themselves (Bishop et al., 2003, p. 211). Concordantly, that all the participants didn't make any effort for the sub-dimension of the rationalism, encouragement of the students to discuss, when mathematical values were examined in ideological dimension, was observed. Only T2 created context for whole class discussions time to time. It can be concluded that only T2 among the participants, even though it depends on the course content, reflects all the values adopted in classroom practices and struggles for placing value duals in classes equally. Based on this result, T2 constituted a kind of personality those value duals at an individual level even if not all the sub-domains of values. This result indicates that the teacher reflects mathematical values on his / her practice even though he is not aware of them. This can be presented as a further research recommendation on the underlying causes and the practical implications of the awareness of the values adopted.

T5 adopted proving which is a sub-dimension of rationalism value. He thinks that in concordance with the previous studies (Güler & Dikici, 2012; Köğce, 2012), proof increases the permanent learning in mind. In addition, he emphasized how proving leads to figure out the emergence of rules and formulas. These findings indicated that T5 is aware of the benefits of mathematical understanding via mathematical demonstration. T1 and T4, hereby, considers that those mathematical proof will result in confuse in students' minds (Köğce, 2012). Results for the rationalism value pointed out that participants adopted this value at individual level, but, interpreted also it as a must to adopt at societal level. Moreover, results showed that participants didn't adopt some indicators of the rationalism value, while they couldn't reflect some others. Based upon the research results, we can conclude that participants should improve themselves in designing the encouraging contexts for critical thinking, queries and induction and they should be trained about the aim and importance to set forth hypothesis including conflict and contradictions among the students.

In concordance with the previous studies (Bishop et al., 2005; Bishop, 2008a, 2008b), according to the results in sentimental dimension of the values, participants adopted control value and reflected at all levels of the courses in classroom practices. They paid attention to '*development of the skills and practices for the fluency of the operations*' sub-dimension of control value, which is parallel with the previous studies (Bishop et al., 2005; Bishop, 2008a, 2008b). This results in because of the use of importance paid to the sub-dimension '*leading practices of operational fluency and skills development*' of control value of the participants We can infer that this value indicator is resulted from the frequent answer-question technique. In parallel with the study of Wan Ali et al. (2007), participants emphasized mathematical solution by using the formula and symbols. Moreover, participants preferred this dimension instead of the indicators of rationalism value, discussion and negotiation, however, T2 and T5 reflected also mystery value in practices where this dimension reflected. These results are the most obvious proofs that control value is reflected at the pedagogical level in classroom practices. This situation necessitated the adoption of control value and research on the resoncs for reflection. Therefore, similarly, we can say that which practices are required in order to gain other mathematical values at the pedagogical level or what to be changed in the application is an important research field. All participants used daily life associated problems to increase the interest and motivation to reflect progress value which is available in complementary dual of the control value. We can evaluate this situation with progress and control values as being the complementary.

Participants adopted control values while reflecting sub-dimensions such as '*explanation for the demonstration or symbol use*', '*explanation of the relationship among notions*' and '*offering suggestions for problem situations*'. In addition to this, participants emphasized the significance of readiness level in the reflection of each dimension according to progress value. As a result of this, while reflecting the objectivism, openness, mystery and progress values, participants gave also place to control value in practices. In other words, it can be inferred that value categorizes at ideological, individual and social dimensions influence each other in the value systems of the individuals.

The application of control and progress values together in classroom practices emerged significant results. One of the attention-grabbing of the results was that participants couldn't foresee about the readiness level of the students since they were themselves reminding the pre-learning of the participants in general. Thereby, this occurred deficiencies in the instructional planning and reflection of the values since they didn't have sufficient information about the readiness level. Thus, we can say that participants avoid of designing practices for '*giving place for the discussion and mathematical demonstration*' of rationalism, '*encouragement of symbol and model use'* of objectivism, '*explanation of the applicability of mathematics for daily life problems'* of control values. This result stems from the belief of teachers about the students' deficient level of readiness. So that, instead of designing context for expressing their opinions, they demonstrate how to explain the mathematical knowledge.

When examined the mathematical values at sociological dimension, results support the findings of the previous studies conducted at different cultural contexts (Bishop et al., 2005; Bishop, 2008a). Even though openness value was observed in classroom practices, participants didn't reflect openness and mystery duals frequently enough. The sub-dimension '*giving opportunity to students to express their opinions in clear and open way*' was observed the most frequently in T2 and T5's practices. On the other hand, other participants preferred students' explanation of individual opinions at the sub-dimension of rationalism by stating that it is not possible every time to give such an opportunity. This sub-dimension emerged during the question and answer method use in practices. Additionally, participants limited the students' freedom of expression at the introduction and explanation stages. This result is consistent with results of VIMT [Values In Mathematics Teaching] project (Bishop et al., 2003). In other words, we can state that teachers don't emphasize enough on the openness value which is the outcome of mathematical knowledge and democratic education and on the mystery value of mathematical knowledge which is required to explore, proof or verify. Based on this result, to design of a democratic classroom environment and have students gain strong aspects of maths such as legibility to prove and verify by the teachers, more research is required about the sub-dimensions of opennes which are not adopted by the teachers.

It can be concluded that participants didn't pay attention to mystery value in classroom practices in contrast to the research on mathematical values by Bishop (2008b). Teachers who reflected the mystery value most frequently, T2 and T5, reflected the openness and rationalism values together. Those participants gave place to mathematical puzzles, furthermore, struggled to create curiosity and interest through unexpected results. In the light of the results found, it is thought that teachers had the mystery value at individual level, however, they should give more importance to the activities emphasizing the significance of the mystery value in syllabus to reflect at academic level.

Participants had problems in design of the classroom practices and reflection of the values to those applications. For example, all the participants found it important to '*associate the mathematical knowledge to daily life*', but, they couldn't apply this for every learning outcome existing in curriculum. In consideration of these results, we can conclude that participants didn't examine the curriculum and take the benefit of the online sources. Another significant point pointed in the current study, concordantly with Seah et al.'s (2001) study, participants aren't aware of the mathematical values at all. Therefore, ineffective reflection of the values in classroom practices may stem from the deficient awareness of the teachers. This situation is out of the teachers' control. Meanwhile, meaningfulness of the learning outcomes are effective on the reflection of the values (Katılmış, Ekşi, & Öztürk, 2011). Because of this, it would be beneficial to take training for values and values education and learning outcomes. So, via the progress value, teachers might gain a further understanding that aim of the education program is not only teaching, but also, deepening of the students' opinions or understanding for previous knowledge. Participants indicate to curriculum, time allocated to achievements, readiness levels of students, and test-oriented assessment and evaluation approaches as the source of their inability to reflect the values they possess, the reasons why the values adopted in the future researches are not reflected in the applications can be figured out by a detailed research. The contribution of our study to these studies will be a recommendation to focuse on why this result comes from the adoption of individual values at the individual level and why it is not adopted at the social or institutional level.

It was seen that T4, who graduated from Education Faculty, paid attention to objectivism and control values, but, reflected openness and mystery relatively less. T2, who graduated from Department of Mathematics, made effort to reflect value duals equally frequently. However, T1, who graduated from Department of Mathematics, reflected rationalism, progress and mystery values rarely. This situation, contradicts with the previous studies (Durmuş et al., 2008; Demir et al., 2012) claiming that this might be result of the undergraduate education. Values existing in the nature of the mathematics as a discipline are gained from the pedagogical accumulation of the teachers (FitzSimons et al., 2001). In concordance with this, it might be more reliable to compare pedagogical and field related education at the faculties in the emergence of those values. Furthermore, in consideration with the effect of values of the class interactions (Bishop et al., 1999), we can say infer that class interactions influence the reflected values on classroom practices in the light of the previous studies (Bishop, 2008c; Clarkson et al., 2000; FitzSimons et al., 2001) and findings. Readiness level of the students, allocated time for learning outcome, potential of the student and student needs can be evaluated within the scope of class interactions. In addition to the results, it was interesting that T1 whose occupational seniority level is 2 years and T5 whose occupational seniority is 26 years stated similar opinions and similar observations were held from their classes. This situation weakens the effect of seniority level variable in the reflection of the values on practices. These results point out the possibility that individual differences effect the values gained. Thus, it is found worth to search about the factors at the individual level. These factors may include school and classroom culture, past experience of the teacher, pedagogical and field knowledge, and beliefs about teaching and learning mathematics.

Shortly, it would be beneficial for the teacher to improve themselves for the values and reflection of the values in terms of increasing awareness and designing applications full of value in classroom practices. In this period, beside of the gain of values by teachers at individual level, curriculum designers should also adopt and improve at instutional level. For this, while educating awareness of value and value education, awareness of moral values as well as philosophical and sociological aspects of the field can be presented as a recommendation to examine classroom observation videos and course materials from the perspective of values. In accordance with this suggestion, why teachers cannot reflect the mathematical values at the individual level and what the necessary variables are to do this, what values the pre-service teachers have and how to have the pre-service teacher gain those values are found significant seek further.

Acknowledgements

We would like to thank to TUBITAK for providing academic scholarship support for this research.

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