The Primary School Students' Pattern Seeking Process In the Spreadsheet Environment*

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Abstract

The process of pattern generalization is essential to the transition period from arithmetic to algebra. The spreadsheet is a technological tool that is effective in mathematics education and supports this process as well. The aim of this study is to investigate the pattern generalization process of primary school 6th-grade students (aged 11-12), who progress through arithmetic to algebra, within the framework of algebraic reasoning in the spreadsheet environment. Therefore, the student data on two pattern questions, one linear and one non-linear, was analyzed. The study found that the students used recursive and explicit strategies with the help of instrumented technics in the spreadsheet environment, and that the spreadsheet environment functioned as a bridge for students' transition from verbal expression of the pattern to its algebraic expression, supporting algebraic reasoning objectives.

Keywords: Spreadsheet, linear and non-linear patterns, generalization strategies, 6th grade.

Introduction

The study of patterns and relationships is regarded, and involved, as one of the basic topics in school mathematics curriculums of many countries. NCTM (2000) considers 'understanding patterns and relations' to be one of the standards of algebra in all grades of primary education, and states that students between 6th-8th grades should be able to generalize patterns. Similarly, after a program change in 2005 in Turkey, the study of patterns is paid close attention to, and by modelling the number patterns, generalizing the pattern rule and expressing it with characters by students are seen as basic skills (Ministry of National Education, 2009).

Research shows that patterns help students form a mathematical relationship, particularly a functional one, adopt to algebra, and develop problem-solving strategies (Hargreaves et al. 1998; Mor et al. 2006), and that making generalizations about arithmetical ideas in number patterns eases forming algebraic relationships (Tall, 1992). Moreover, 'observation, formulization, examination and visualization of patterns and relations' is regarded as one of the mathematical activities essential to enhancing algebraic reasoning (Dekker & Dolk, 2011). Therefore, algebra is considered to be the language of patterns, quantitative relationships and thus generalization (Usiskin, 1995; 1999). Generalization is a requirement in examination of patterns (Jones, 1993:27). While Zazkis & Liljedal (2002) state that patterns are the heart and the soul of mathematics, Mason (1996) calls generalization the vein of mathematics, adding that it is one of the basics of algebra. Not only is recognizing and generalizing patterns vital in mathematical reasoning but it is a beneficial way in algebraic reasoning as well (Mor et al. 2006). Given that algebra is a means of expression of generalizations, the discovery of patterns as a start is integral to algebraic reasoning (Vale & Cabrita, 2011).

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Most national and international research on patterns was carried out to examine the pattern generalization (Akkan & Cakiroglu, 2012; Bas et al., 2011; Bishop, 2000; Hargreaves et al., 1999; Lannin, 2003; Sasman et al., 1999; Stacey, 1989; Tanisli & Yavuzkoy Kose, 2011; Yesildere & Akkoc, 2011). In these studies, students of different grades were asked to do number, shape, linear and non-linear pattern problems in the paper-and-pencil environment, and thus what strategies they used while generalizing the patterns were identified. For instance, while Hargreaves et al. (1999) examined linear number patterns with 1-5th grades students, Bas et al. (2011) studied linear shape patterns with 9th grade students and Tanisli and Kose (2011) studied with teacher candidates. Akkan and Cakiroglu (2012) and Sasman et al. (1999), on the other hand, investigated linear and non-linear patterns with 6-8th grades students, while Yesildere and Akkoc (2011) studied with mathematics teacher candidates. Even though the researchers used different terms for strategies, it is seen that they mentioned the same strategies, which can be listed as modelling, multiplication of the difference, extension of the whole, prediction and testing, contextualizing, explicit, and recursive (Bishop, 2000; Hargreaves et al., 1999; Lannin, 2003, 2005; Stacey, 1989). When the findings of the studies are investigated, it is clear that the strategies that students select more than others are as follows:

- Recursive: Recognition of a relationship between independent variables.
- Explicit: Recognition of a relationship between dependent and independent variables.
- Extension of the whole: Extension of the next term to determine a far term.
- Multiplication of the difference: Multiplication of the term number with the common difference in the pattern.

It was also found that essential to algebraic reasoning, the process of generalizing patterns, which is, in other words, the transitional period from mathematics to algebra, was not easily realized by some students (Sasman et al., 1999). In such circumstances, it is imperative that teachers pay utmost attention to varied strategies for making generalizations, encourage students to use abstract strategies, and conceptually enhance the student comprehension of the reasoning behind particularly the explicit strategy expressing the relationship between the term and its reference (Akkan & Cakiroglu, 2012; Cai & Moyer, 2008).

It is also stated in studies on algebra that learning environments can be supported with the aid of technology during both algebra teaching period and transitional period from arithmetic to algebra (Abramovich & Nabors, 1997; Heid & Blume, 2008; Tabach et al., 2008). One of these technological aids is the spreadsheet, which is not created mainly for educational purposes yet can be commonly used in mathematics education starting from primary school.

The potential of the spreadsheets in algebra education has been investigated by many researchers (Ainley, Bills & Wilson, 2005). Working with the spreadsheet is one of the ways that help students forward from a non-algebraic approach to an algebraic one (Jones, 2005). Compared to the paper-and-pencil environment, students can learn better to express mathematical relationships by using a symbolic language in the spreadsheet environment (Tabach, 2011). Thus, spreadsheets support the transitional period from arithmetic to algebra (Bills et al., 2005; Dettori et al, 2001; Rojano, 1996). Wilson et al. (2004) state that the spreadsheet environment plays an important role in fostering students' making generalizations, supporting paper-and-pen activities. Furthermore, Rojano (1996) supports the idea that spreadsheets promote inductive thinking skills, adding that students in the study gained algebraic proceeding process as a result. In other words, the language of the spreadsheet emerges as a useful mediator during the transitional period from the examination of patterns to the generalization of them (Abramovich & Nabors, 1997).

The Framework of Algebraic Reasoning in the Spreadsheet Environment

According to Herbert and Brown (1997), algebraic reasoning is the revelation of information out of a problem situation, the representation of this information mathematically (in diagrams, tables, charts, equations, etc.), the interpretation and implementation of the findings for a new problem situation that is the same as or related to the examined one, and the use of mathematical symbols and tools in further analyses of different problem situations. The examination of pattern problems is interpreted as the specific component of this kind of algebraic reasoning framework shown in Figure 1. The investigation process of pattern problems consists of three phases: (1) seeking the pattern, (2) recognizing/ identifying the pattern, and (3) generalizing the pattern. While seeking the pattern is the examination of clues in the problem situation given, recognizing/ identifying the pattern is a mathematical analysis. During this phase, multiple representations (mathematical terms, diagrams, tables, charts and equations) can make it easier for students to discover the pattern. Generalizing patterns is then the interpretation and implementation of the information obtained during the first two phases for far and n. terms of the pattern. Testing the term values, identifying the functional relationships, coming out with an appropriate formula for the problem situation, and interpreting and implementing it into new situations can all be done during the generalization phase. As a result, students can understand the power of algebraic reasoning through generalizing patterns (Herbert & Brown, 1997).

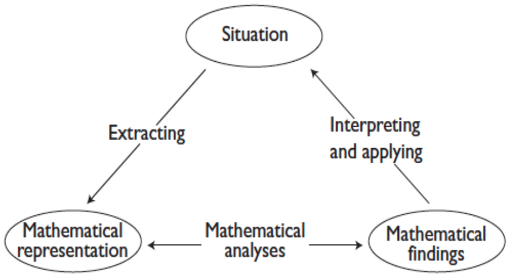


Figure 1. Algebraic Thinking Framework (Herbert & Brown, 1997, p.124)

The studies have established that the structure of the environment undergoes change with the integration of technology in the learning environment, and it is necessary that a new approach be adopted after reviewing the didactic, pedagogic and epistemic ones (Artigue, 2002; Lagrange, 2000). Lagrange (2000) mentions that the structure of problem searching and solving techniques will alter in a technological environment, and identifies these techniques obtained with the help of technological aids as the concept of 'instrumented technique'. He also indicates the importance of instrumented techniques in mathematics education for students' conceptual learning.

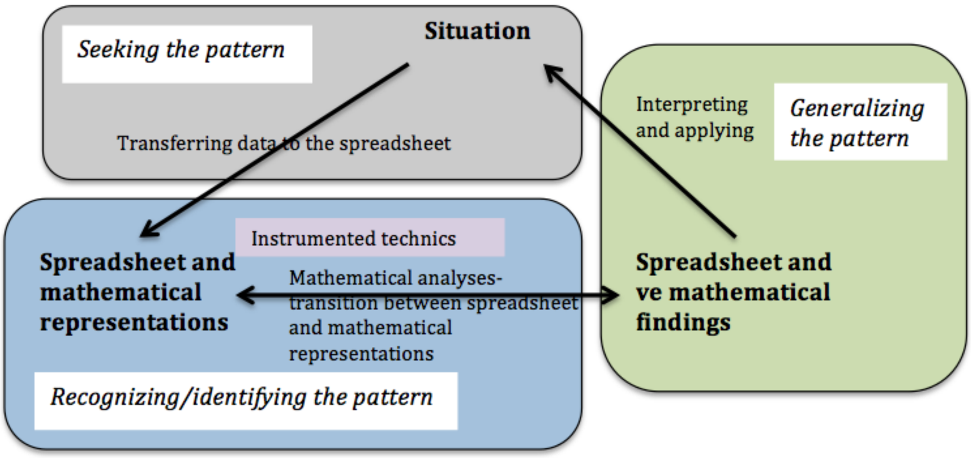


Figure 2. Algebraic Thinking Framework in the Spreadsheet Environment

Taking into consideration the instrumented techniques during the examination of algebraic problem situations in the spreadsheet environment, we can interpret the algebraic reasoning framework that Herbert and Brown designed as follows (Figure 2): While seeking the pattern, the problem situation is examined, and the information is transferred onto the spreadsheet. During the mathematical analysis at the recognizing/ identifying the pattern phase, the representations on the spreadsheet and mathematical representations are used (tables, the cell references, formulas based on the cell references, etc.) This phase also includes different strategies that students use by utilizing representation shapes. Different instrumented techniques which students will make use of come out in both seeking and recognizing/identifying the pattern phases. The generalizing the pattern phase is the examination of the findings obtained in the spreadsheet environment, the transition from the spreadsheet representations to mathematical ones, and arriving at mathematical findings. In this way, generalizing the pattern rule for the n. term will be made.

This study was conducted in order to investigate the 6th grade students' pattern seeking and generalizing process in the spreadsheet within the framework of algebraic reasoning in the spreadsheet environment.

Method

The data used in this study was obtained from the long-term research designed as a teaching experiment within the scope of a master's thesis (Turan, 2013). The teaching experiment is a dynamic method designed primarily to discover and understand students' mathematical strengths (Steffe & Thompson, 2000). In the study, students were first provided with a general definition of the spreadsheet, and then some simple problems where students worked individually or in pairs were involved. After that, the research questions about patterns, which are investigated in this paper, were presented.

Participants

The participants of this study are 15 6th-grade students (aged 11-12) at a state primary school in the city centre of Eskisehir. The students were introduced to the spreadsheet for the first time, and some of their math class hours were done in a computer laboratory. Due to the limitation on the number of computers in the laboratory, 11 students worked individually, and 4 students worked in pairs.

Data Analysis

The computer screens of students during practice sessions were captured with Camtasia Studio 7 software, and students' worksheets at the end of sessions were saved. Besides, a general view of the classroom was videotaped, and the teacher's whole-class or group conversations were recorded with a personal microphone. In this way, thanks to the screen recordings with Camtasia Studio 7, the pattern seeking phases of each student on the spreadsheet and the techniques they used were easily monitored after the practice sessions, and the phases for each student were documented first. Next, these documents were re-examined together with the voice recordings (dialogues among other students or between the teacher), worksheets and the data in the classroom videotape, and then all the information about each student during sessions was finalized. Last, the content analysis of the data from each student/ student pair out of quantitative research methods was made.

Instruments

a) The spreadsheet: One of the spreadsheets widely used, the Microsoft Excel Workbook, was used in this study. Appearing in the market in the 1980s first, the spreadsheet is interactive software that transfers large column-and-row pages (*worksheets*) used to record a workplace's account into a computerized environment. The basics of the spreadsheet that must be known are the following:

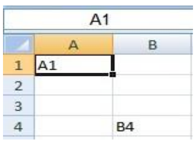


Figure 3. Cell

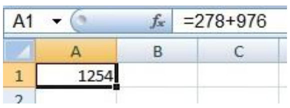


Figure 4. Mathematical operation

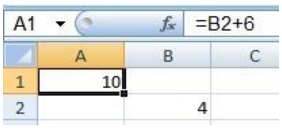


Figure 5. Operation with cell address

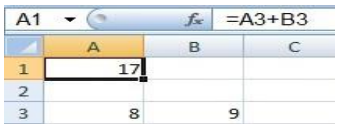


Figure 6. Operation with multiple cell addresses

Cell: The columns on the spreadsheet are widely shown in alphabetical letters, and the rows in numerical characters. Each rectangular box in a row worksheet is referred to as a cell. A cell is the intersection point of a column and a row. To keep track of where data is stored, each cell has a cell reference, also called a cell address, consisting of the column letter and row number of where the cell is located. For instance, in Figure 3, A1 cell is where column A and row 1 intersect. Similarly, the name of the cell where column B intersects with row 4 is B4. Students first need to have the ability to determine the cell references. Cell references are the main representations on the spreadsheet, which can be regarded as the x variable representation in algebra.

Executing Mathematical Operations and Using Equal Sign: In order to execute a mathematical operation in a cell, the operation in the cell must be entered by using the equal sign. For example, when an operation as =278+976 is written into a cell and the 'enter' key is clicked on, the result of the operation, 1254, is obtained, and the operation is displayed in the formula bar at the same time (Figure 4).

Making a Formula: When a cell reference is used in another, for instance in cell A1 =B2+6 (Figure 5), or several cell references are used in another, for instance =A3+B3 (Figure 6), formulas are made and thus mathematical operations can be executed.

Drag Option: One of the basic uses of the spreadsheet is the copy function. In order to copy, the cell(s) selected are dragged with the mouse through rows or columns. Depending on the content of the cell, different results may be obtained from dragging. To illustrate, if there is a number in a cell, selecting and dragging that cell copies the same number into the other cell(s) (Figure 7). When at least two adjacent or top and bottom cells, whose values are, for instance, a and b, are both selected and dragged, the spreadsheet creates a series in b-a values (Figure 8). On a spreadsheet, a series of substitution (day, month, etc.) can be obtained automatically. When a formula in a cell is selected and dragged, this formula is copied into the new cells, keeping the mathematical relationship. For instance, Figure 9 shows the obtained formulas on the spreadsheet after a formula is entered into another cell, selected and dragged down.

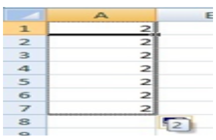


Figure 7. Dragging one number

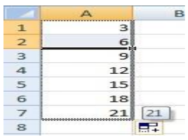


Figure 8. Dragging two numbers

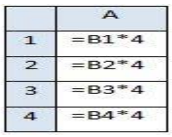
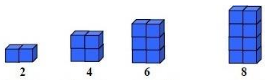


Figure 9. Dragging a formula

b) Activities: In this study, two activities in the second stage of the research designed as a four-week teaching experiment were investigated. Prior to these activities, which in other words means at the beginning of the teaching experiment, researchers introduced the spreadsheet to the students briefly. Then, the stages of the teaching experiment were followed. Eight different activities, totally 8 hours, were presented to the students in the first two weeks. Six of them were aimed at the introduction to the spreadsheet (using the menu bar, cell references, four operations, four operations using cell references, drag option, opening a worksheet), which is the first stage of the teaching experiment, and the other two were aimed at the second stage of the experiment, which is about patterns and relations (finding the rule in number patterns on the spreadsheet, finding the rule depending on the number of terms in the pattern).

During the third week of the teaching experiment, students were asked to do two pattern problems for two hours (Figure 10). In these two problem activities, students were expected to create worksheets by entering the information of the patterns in the spreadsheet environment, to find the pattern rule and convert it into the spreadsheet formula, and to generalize the pattern and present the general n. term formula. The first problem question includes a linear number pattern modelled with cubes, while the second one includes a non-linear shape pattern. The students were asked to find first the next terms then the n. terms of the patterns, and finally to interpret the n. terms. The rule for the n. term in the first pattern is nx2, and that of the second pattern is nxn.

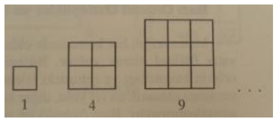
CUBES



You see a pattern "2 4 6 8..." with cubes. Examine the pattern and convert the information into a table in Excel.

How many cubes are required for the 5^{th} term? ____
How many cubes are required for the 10^{th} term? ____
How many cubes are required for the 25^{th} term? ____
How many cubes are required for the 100^{th} term? ___
Write the formula if you found one __________
How many cubes are required for the n. term? ____

SQUARES



Examine the pattern and convert the information into a table in Excel.

How many squares are required for the 8^{h} term? ____
What formula do you need to enter for this term?
How many squares are required for the 30^{h} term? ____
What formula do you need to enter for this term?
How many squares are required for the 200^{h} term? ____
What formula do you need to enter for this term?
How many squares are required for the n. term? ____

Figure 10. Pattern problems

The study started with the students' examining the pattern in the paper-and-pencil environment. The students tried to find a relationship between the terms by counting the models, which is followed by a class discussion about how the pattern proceeds. After such an introduction to the problems with paper-and-pencil instruments, the students were asked to switch to the spreadsheet environment and work on the spreadsheet in order to find the near and far terms together with the pattern rule. During this period, the students transferred the information about the pattern onto the spreadsheet and used the functions of the spreadsheet (dragging, entering a formula). The students' worksheets were monitored, and when they had difficulty, the teacher asked whole-class questions thus guided them with whole-class discussions about finding the pattern rule and entering a formula. At the last stage of the activity, the students were asked to describe the pattern rule, which they had found in the spreadsheet environment, on the spreadsheet in algebraic terms, and they were asked questions about the algebraic terms and alphabetical symbols they used thus were made to interpret.

Findings

The distributions of the students according to the environment they worked in and the strategies they used are provided in the table below.

Table 1.
Strategies Used

	Activity 1			Activity 2		
Strategies used	**SE**	**PP**	**SE < -- >PP**	**SE**	**PP**	**SE < -- >PP**
Recursive	3	-	-	-	-	-
Explicit	4	-	-	9	1	-
Recursive + Explicit	4	-	4	-	1	3
None					1	
TOTAL	11	0	4	9	3	3

SE: Spreadsheet Environment, PP: Paper-and-Pencil Environment

As can be seen in Table 1, the majority of the students worked in the spreadsheet environment, yet some students worked in both the spreadsheet and paper-and-pencil environment. Two strategies were used in order to generalize the pattern: recırsive and explicite. While, in activity 1, there is an equal distribution of the strategies used, the explicit strategy was used in activity 2. A detailed analysis of the students' processes of generalizing the pattern is presented below.

Activity 1

Seeking the Pattern: Students started with examining the problem situation. It was observed that the students were able to determine the 5th term easily after examining the model and the number pattern given. The instructions asked students to transfer the problem situation into the spreadsheet environment. However, the students were observed to ignore the instruction and keep working in the paper-and-pencil environment, at which stage the teacher got involved by reading the instructions aloud in class and guided the students to work in the spreadsheet environment.

	A	B
1	2	
2	4	
3	6	
4	8	
5	10	

Figure 11. Ufuk's worksheet

	A	B
1	1	2
2	2	4
3	3	6
4	4	8
5	5	10

Figure 12. Rabia ve Nesrin's worksheet

	A	B
1	1	=A1*2
2	2	=A2*2
3	3	=A3*2
4	4	=A4*2
5	5	=A5*2

Figure 13. Formula with Explicit Strategy

Recognizing/ Identifying the Pattern: During this phase, the students created two tables in the spreadsheet environment, as seen in Figures 11 and 12, and transferred the data about the near terms that they obtained during the preceding phase. As in Ufuk's worksheet in Figure 12, some students created a table by entering only the term values and took the row numbers of the spreadsheet into account for the number of terms in the pattern. Some students, on the other hand, created a two-column table consisting of the number of terms in the pattern and term values, as in Rabia and Nesrin's pair worksheet shown in Figure 12, and transferred the data onto the spreadsheet.

It is observed that in the spreadsheet environment, all the students were able to determine the first term values with drag option. 8 of these students, who created their tables by dragging, started to look for a formula by which they could obtain the pattern in the spreadsheet environment. It was found that the students compared the pattern values they obtained with the drag option with the patterns they obtained with the formula in order to check whether the formula they made was accurate or not.

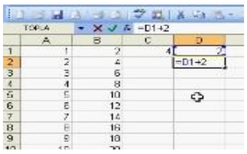


Figure 14. Formula with Recursive Strategy

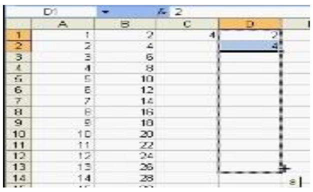


Figure 15. Dragging the number and the formula together

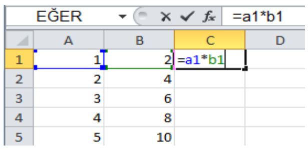


Figure 16. Rabia and Nesrin's worksheet

4 of these students (Zuhal, Seyfullah, Ilkay, Eray) used the explicit strategy to calculate the terms of the pattern. These students created the spreadsheet formula based on the number of terms in the pattern, entered it into the cell and obtained the other terms by dragging the formula. For instance, the formula =A1*2 was entered into cell B1 and copied down (Figure 13). Other 4 students (Fatmanur, Ufuk, Nur, Gokce) tended towards the recursive strategy and made formulas in the spreadsheet environment by adding 2 to the preceding values. To illustrate, Ufuk entered the number 2 into cell D1 and then the formula =D1+2 into cell D2 (Figure 14). Other 3 students made the same formula based on the column they worked on. After this step, what students need to do was to copy the formula by dragging down. When the worksheets of the students were examined, it was seen that 4 students were not able to determine the pattern targeted. When the video recordings were examined

later on, it was noticed that these students were not able to drag properly. For instance, it was determined that Ufuk selected D1 and D2 (two cells with number and formula) and dragged instead of dragging the formula in cell D2 (Figure 15). When the students saw the pattern they obtained by dragging the formula and the one they obtained by dragging the terms, they concluded that the formula was incorrect and started to look for a new one thereby tending towards the explicit strategy. Therefore, they entered the formula in the tables as in Figure 13 and obtained the pattern with the drag option. Although the formula these students made with the recursive strategy was correct, they were not able to get the result due to dragging the formula with the number as in Figure 14, but they did not have any trouble dragging with the explicit strategy as both cells had the formula then.

Unlike other students, Fatmanur, Ufuk, Nur and Gokce entered the formula into a different column than the one they worked on. After that, by dragging the formula, they checked whether or not the column was the same as the other one and thus made sure that the formula they entered was correct. It was seen that the students created the columns D and B so as to compare and check.

Three students (Azat, Rabia, Nesrin), on the other hand, were observed to answer the problems with only the patterns they obtained by dragging the first terms but to fail in search for a formula for the pattern. The students created the number of terms in column A and the necessary number of the cubes in column B with the drag option. Rabia and Nesrin entered the formula =A1*B1 into cell C1 and selected and dragged cell C1. Those students who considered the rule to be twice as the number of the term made the formula this way as the number 2 was found in cell B1, but they failed to obtain the same pattern because of the change in the formula they wrote by dragging (The formula in C2 becomes =A2*B2) (Figure 16). These students were not able to progress through this pattern from then on.

It was also seen that 4 students (Melis, Alperen, Elifnur, Gurkan), who worked in both the spreadsheet and paper-and-pencil environments, used not only the recursive but also the explicit strategies. These students first found the required term values by using the recursive strategy with the drag option. Then, they did not enter any formulas onto the spreadsheet and switched to paper-and-pencil environment. Among these students, Melis and Elifnur expressed the pattern rule with explicit strategy on their worksheets by using the spreadsheet representations. For example, Elifnur wrote the formula =e1x2 for the 5th term, and similarly, Melis wrote the formula =A5x2 (Figures 17 and 18).

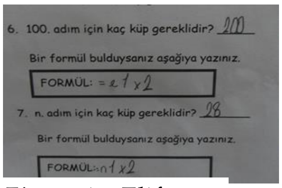


Figure 17. Elifnur

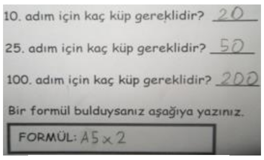


Figure 18. Melis

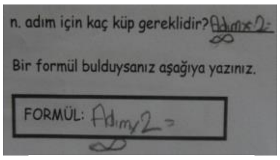


Figure 19. Alperen

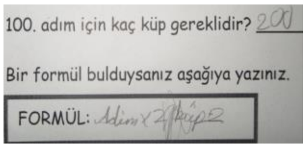


Figure 20. Gürkan

It was, on the other hand, also observed that the other two students (Alperen and Gurkan) wrote an *algebraic formula* (Figures 19 and 20). Whereas Alperen expressed the pattern rule by using the explicit strategy, Gurkan expressed it by using both explicit and recursive strategies.

When the worksheets created in the spreadsheet environment were examined, it was seen that the mathematical representations were replaced by the spreadsheet representations depending on the working environment during this phase, where problem situations are transferred into mathematical representations and a mathematical analysis emerges.

Generalizing the Pattern: In the spreadsheet environment, far terms can easily be found with the drag option after calculating the near terms. As an illustration, 14 students who created the pattern by using the drag option after entering the first values were seen to provide the number of the cubes required for the far terms at ease.

The students developed ideas about generalizing the pattern during the period of entering a formula for the near term and made comments on the pattern rule.

Teacher: *How many cubes are required for the 5th term?*

Melis: *10*
Teacher: *What about the 25th one?*
Alperen: *Sir, it's already twice as many.*
Teacher: *Twice as many as what?*
Alperen: *The number of term.*
Gurkan: *Twice as many plus 2 more.*

As can be seen in the excerpt, the pattern rule was expressed in accordance with both the explicit strategy (Alperen) and the recursive one (Gurkan).

The last step of the generalization stage during the pattern-seeking phase is that the students interpreted and applied the mathematical data they obtained.

For the first activity in this phase, students were asked to express the generalization obtained with the spreadsheet representation about the 100. term for n.term. Thanks to this pattern problem, students were introduced to the use of alphabetical symbol for the first time. It was observed that 10 students were able to express the pattern rule for the n. term and switch from the spreadsheet representation to the algebraic one. For instance, Figure 21 shows the table Seyfullah created in the spreadsheet environment (one created by dragging the formula), and Figure 22 shows how he progressed through the n. term on the paper-and-pencil environment.

	A	B
97	97	=A97*2
98	98	=A98*2
99	99	=A99*2
100	100	=A100*2

Figure 21. Seyfullah's worksheet on spreadsheet

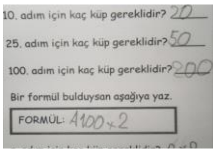


Figure 22. Seyfullah's worksheet

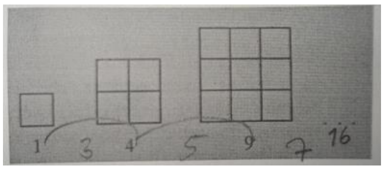


Figure 23. Seeking a recursive relationship

In addition, some misconceptions about the concept 'n. term' were observed. To illustrate, 3 students matched the n. term with the column name on the spreadsheet. The students took the column N as the 14th column and regarded it as the number of the term and thus found the value of the n. term 28. One student interpreted it as the letter following m. At the end of the activity, however, it was observed that these students were able to interpret the n. term.

Teacher: *What is n you're talking about?*
Melis: *The number of term.*
Teacher: *What numbers can we write instead of n?*
Alperen: *Infinite.*
Teacher: *Infinite?*
Alperen: *I mean any number.*
Melis: *3, 5, 7, 9, 11, 13.*

Activity 2:

Seeking the Pattern: 14 of the students started with the examination of the model provided. While some of them were examining the relationship between the number of the term and the term value, some others tried to find a recursive relationship by looking into the difference between the values (Figure 23). The comments of the students were, as an example, are as follows:

Ilkay: *It increases to 3, to 5, then will increase to 7, then to 9. 25!*
Ufuk: *For the first term, 1 multiplied by 1, for the second 2 multiplied by 2, for the third 3 multiplied by 3.*

One student (Seyfullah) was observed to transfer the data on the paper onto the spreadsheet without the need to examine it in the paper-and-pencil environment and obtained a pattern with the drag option.

Recognizing/Identifying of the Pattern: 4 students (Alperen, Melis, Nur and Gokce in pairs) created two tables, a one-column and a two-column, as in the first activity and transferred the data into the spreadsheet environment. During this activity, it was observed that 9 students looked into the other terms only in the spreadsheet environment while 4 students in both the spreadsheet and paper-and-pencil environments, and that 1 student was not able to progress through the activity.

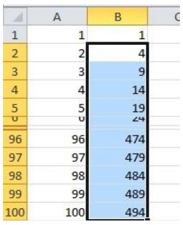


Figure 24. Seyfullah's worksheet on the spreadsheet

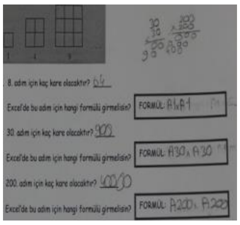


Figure 25. Alperen's work on the paper

What 6 students attempted in order to create the pattern was to copy the first two or more terms of the pattern by dragging. Even though this strategy worked with linear patterns, as in the first activity, it did not work out with such non-linear patterns as this activity and resulted in a different pattern. 5 students noticed that they did not get the correct square numbers by dragging the first two terms. These students, who realized that the drag option would not work here, went on to make a formula after creating the term numbers. Nevertheless, 1 student (Seyfullah), who considered that the number pattern obtained by dragging the first terms of the spreadsheet was the expected pattern, answered the other questions quickly in accordance with the pattern the spreadsheet provided (Figure 24).

Teacher: *How many squares are required for the 100th term?*
Seyfullah: *494*
Teacher: *For the 100th term?*
Seyfullah: *Yes.*
Teacher: *How come 494?*
Seyfullah: *....!? I must have dragged incorrectly...*
Teacher: *What did your write for the formula?*
Seyfullah: *I haven't yet.*
Zuhal: *He just dragged and made a mistake.*

5 of the students (Gurkan, Alperen, Eray, Seyfullah) who first tried to do a mathematical analysis of the pattern in the paper-and-pencil environment focused on the difference among the square numbers by using a recursive strategy.

Teacher: *It goes as 1, 4, and 9. Then16, what rule is followed?*
Ilkay, Eray and Gurkan: *It goes up to 3, then to 5, then to 7, and then to 9..*
Seyfullah: *Plus 3, plus 5, plus 7… so by 2.*
Alperen: *So it always increases by 2.*

Therefore, it is clear that Gurkan, Eray and Ilkay evaluated the difference among the numbers of the pattern, whereas Alperen and Seyfullah evaluated the difference of the difference among the numbers of the pattern. 5 students who were not able to make the formula that would provide the 100th term out of the difference among the numbers of pattern left the recursive strategy they followed and adopted an explicit one. One of them (Alperen) went back to work in the paper-and-pencil environment, executed some mathematical operations connected to the explicit strategy, wrote the rule "the term number x the term number" on his paper, then erased it, and finally wrote the formula that he thought would be correct on the spreadsheet (Figure 25). The other 4 (Gurkan, Eray, Ilkay, Seyfullah) attempted to search for the formula in the spreadsheet environment.

During this stage, where students converted the rule they expressed in words in the paper-and-pencil environment (the term number x the term number) into the spreadsheet formula, they were observed to make some mistakes (Nesrin, Rabia, Ufuk, Seyfullah). The common mistake was that the formulas entered as =A1*1, =A2*2 were selected and dragged, as shown in Figure 26. Although these formulas were correct for the first cell (for instance, because A2 corresponded to the 2nd term in A2*2, the expression became 2*2), they were incorrect for the other terms. While copying these formulas, A values increased as A1, A2, A3, etc., whereas *1 and *2 numbers stayed the same for the other cells (Figure 27). At this point, the teacher guided the students by calling their attention to the change during the copy process. 13 students, including Gurkan, Eray, Ilkay and Seyfullah, who previously worked in the paper-and-pencil environment, used the explicit strategy, entered the formula =A1*A1, copied it into the other cells by dragging, and found the pattern required (Figure 28).

	A	B
1	1	=A1*1
2	2	=A2*2
3	3	
4	4	

Figure 26. Formula work on the spreadsheet table

	A	B
1	1	=A1*1
2	2	=A2*2
3	3	=A3*1
4	4	=A4*2

Figure 27. Dragging the formula

	A	B
1	1	=A1*A1
2	2	=A2*A2
3	3	=A3*A3
4	4	=A4*A4

Figure 28. Dragging the formula obtained from explicit strategy

Generalizing the Pattern: 14 students expressed the pattern rule for the n. term by checking the formulas and switched to these formulas on the spreadsheet. For example, Figure 29 shows the table Zuhal created in the spreadsheet environment (by dragging the formula), and Figure 30 shows how she switched to the n. term in the paper-and-pencil environment.

	A	B
97	197	=A197*A197
98	198	=A198*A198
99	199	=A199*A199
100	200	=A200*A200

Figure 29. Zuhal's spreadsheet

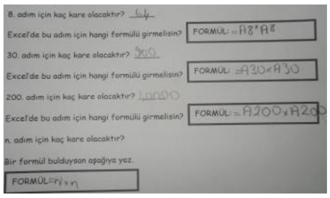


Figure 30. Zuhal's work on the paper

The following are some utterances by students about interpreting the pattern while seeking the formula they would enter into the table.

Fatmanur: We multiplied 4 by 4 for the 4th term. We multiply by 5 for the 5th. We multiply the term number by itself.
Ilkay: We multiply the term number by the term number.
Seyfullah: The term number is multiplied by itself.

While generalizing the patterns, the students were observed to make mistakes related to the technology use. The commonest ones were related to the drag option of the spreadsheet. The students selected and dragged either just one cell with a number to get a series of number or two cells, one with the term number the other with the formula, to copy the formula. In the first situation, a copied series (1, 1, 1, ... etc.) was obtained rather than a filled one on the spreadsheet, while in the second situation, some cells included just formulas and some others just numbers.

Discussion and Conclusion

In this study, which investigates the students' pattern seeking processes with the help of two activities within the framework of algebraic reasoning in the spreadsheet environment, all the students involved except one followed the phases of seeking the pattern, recognizing/ identifying the pattern, and generalizing the pattern.

During the seeking the pattern phase, it was observed that the students focused on the pattern data while transferring the information about the problem situation and created the tables as a result. As Hargreaves et al. (1999) states, insisting on the examination of the pattern is essential to both focus on the activity and forward the next step.

During the recognizing/ identifying the pattern phase, it was observed that instrumented techniques and the spreadsheet representations were involved in the examination process together with a mathematical analysis. Similar to studies by Lannin (2005) and Lannin et al. (2006), explicit and recursive strategies came out in this study, too as strategies students use in the spreadsheet environment.

That only these two strategies stood out might cause a wrong perception at first glance that this spreadsheet environment is in adequate. However, when taken into account the potential of making students acquire these strategies connected to the importance of writing a formula to obtain the spreadsheet pattern or the necessity of the drag option, the requirement of the afore-mentioned spreadsheet environment can easily be understood. Examination by using recursive and explicit strategies will provide the solutions to problem situations (Lannin 2005). The recursive strategy cannot be generalized but is crucial due to emphasizing the common difference in the pattern (Garcia-Cruz & Martinon, 1998). The recursive strategy also enhances the acquisition of the explicit strategy. If students can make a connection between the recursive strategy and the numbers they find, they manage to develop the explicit strategy (Lannin, 2003, 2005; Lannin et al., 2006). The use of the explicit strategy will have an effect on the further development of formal algebra (Stacey, 1989; Swafford & Langrall, 2000). It is suggested that students be taught the reasoning behind the explicit strategy particularly defining the relationship between the term and the term reference (Bishop, 2000). Both strategies help students with the preparations for the perception of the functional relationship and comprehension of the function concept.

It is quite easy to obtain linear patterns with the recursive strategy in the spreadsheet environment by using instrumented techniques. The same result cannot be obtained with the explicit strategy and requires students to be able to write the spreadsheet formula. In this study, similar findings to the ones in Lannin's study (2003) were obtained. For instance, while the majority of the students attained the pattern easily by dragging the number they used for the first pattern, they were not able to use the same instrumented technique for the second pattern and sought to make a formula with the explicit strategy.

On the other hand, the correct use of the instrumented techniques is vital at this stage. Some students did not manage to get the required result on the grounds that they did not use the drag option properly. Their recursive formulas in the first pattern were correct, though, and they related the incorrect result to their inaccurate mathematical analysis and incorrect recursive formula they made. Despite the fact that these students kept the same technical mistake, their finding the correct answer with the help of the pattern formula based on the explicit strategy supported their way of thinking. These students were not able to notice that they used the instrumented technique out of the feedback provided by the spreadsheet. In literature, this situation is defined as the constraints of the artefact (Balacheff, 1994; Guin & Trouche, 1999).

It might be wrong to regard the generalizing phase in the spreadsheet environment as a step following the recognizing phase and a totally different one. Stacey (1989) mentions two generalizations in patterns: near generalization (such as the 20th term), which is to obtain the terms by drawing or counting term-by-term; far generalization (such as the 100^{th} term), which is to obtain the terms beyond the term-by-term approach. For far generalizations (the 100^{th} term, the 1000^{th} term), students need to develop the general rule and use it. 100 and 1000 here play the role of generalized numbers. In the spreadsheet environment, students can easily make the far generalization with instrumented techniques after they can make the near generalization during the recognizing/ identifying the pattern phase. Students make the spreadsheet formulas for the near generalization, which is not so different from generalizing the pattern. In other words, unlike the paper-and-pencil environment, generalization in the spreadsheet environment appears while calculating the near terms. The formula students attempt to make for near terms transforms into the generalization of the pattern and their observation about the same expression for the other terms. In this study as well, it was observed that students who had not progressed through algebra attempted to formulize the verbal rules they expressed for near terms by expressing in the spreadsheet representations, and the majority of them made these formulas.

These findings support the ones Dettori et al (2001) and Ploger et al. (1997) obtained. Dettori et al. (2001) advocates that the use of spreadsheet should not be limited to merely doing algebraic activities, and that using the algebraic language and making explanations with this tool is essential. In their studies with 5^{th}-grade students, Ploger et al. (1997) state that students discover the pattern at ease by transferring the problem situation from the paper-and-pencil environment onto the spreadsheet, and that the spreadsheet is used as an effective tool in order to generalize the pattern.

During the last stage, where students were asked to make an algebraic generalization of the pattern and to state the expression for the n. term, the first answers of the students who were introduced to algebra for the first time bear a similarity to the findings in literature (Kuchemann, 1981 and Kieran, 1992; cited in Akkan, 2009; Stacey & MacGregor, 1997) that alphabetical symbols are assigned numerical values. During the further stages of the study, it was observed that this idea of students was replaced with the idea of "the number representing any term", and students expressed the pattern rule algebraically out of the formulas they wrote on the spreadsheet. Bishop (2000) states that teachers should promote the use of the algebraic language during the process of pattern generalization. As mentioned earlier, the spreadsheet representations might function as a bridge between the students' natural language and the formal representation of algebra (Rojano & Sutherland, 1993; Rojano & Sutherland, 1994; Tabach, 2011).

This study demonstrates that students realize the pattern examination phases at ease and use the instrumented techniques and the spreadsheet representations during this process. Therefore, the use of the spreadsheet should be encouraged during the transition period from mathematics to algebra, particularly in patterns. Spreadsheet activities including the use of recursive and explicit strategies related to the comprehension of the functional relationship should be paid utmost attention. Activities that will improve students' ability to use the algebraic language by means of the spreadsheet might be done. On the other hand, the examination of the patterns in the spreadsheet environment should not be limited to the purpose of supporting patterns. It should be taken into consideration that the spreadsheet has a potential not only for the transition period from arithmetic to algebra but also for the enhancement of algebraic reasoning skills. Including the algebraic reasoning in the spreadsheet environment in teachers and teacher candidates' formation will make a huge contribution to algebra teaching.

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