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Developing Mathematical Competencies in Secondary Students by Introducing Dynamic Geometry Systems in the Classroom

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Abstract

This paper reports part of a design research experiment aimed at studying the influence of Dynamic Geometry Systems (in particular, GeoGebra) in students´ mathematical competencies development. It was conducted in two cycles with four classes at a secondary school. The results show evidences that the software helped students to develop mathematical competencies at different degrees. Geogebra provided students enough support for reaching a good level of performance in competencies related with using tools, managing representations, and problem solving. The software supported students in achieving basic to medium levels in competencies related to reasoning, argumentation and communication. In these last competencies, social interaction proved to be a key factor for students´ progress.

Keywords

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Introduction

Research in new technologies is one of the main focuses of interest to educators and researchers in Mathematics Education, as reflected in the International Commission Mathematics Education studies in 1985 (Cornu & Ralston, 1992) and 2006 (Hoyles & Lagrange, 2010). Initially, in the early 80's, the studies focused on the possibilities of computing and programming. Gradually, in the 90's, and due to new possibilities of representation and calculation of Dynamic Geometry Systems (DGS) and Computer Algebra System (CAS), studies began to consider these technologies as cognitive tools that allow studying mathematics in an interconnected manner. Nowadays, there is a demand for works which explore the use of technology for learning, address methodological issues, and provide assessment methods and instruments (Baki, Güven, Karatas, Akkan, & Çakıroğlu, 2011).

In order to promote and assess mathematical learning in students, there is an international trend to focus on the development of their mathematical competency (Aydin, Sarier & Uysal, 2012; the PISA Project (Organization for Economic Co-operation and Development [OECD], 2003, 2006, 2009). The PISA project notes that:

- An individual who is to engage successfully in mathematization within a variety of situations, extra-and intra-mathematical contexts, and overarching ideas, needs to possess a number of mathematical competencies which, taken together, can be seen as constituting comprehensive mathematical competency. Each of these competencies can be possessed at different levels of mastery (OECD, 2003: 40).

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Various studies show the DGS potential for fostering some of these competencies in students (Aydogan, 2007; Guven & Kosa, 2008; Kurtuluş, 2011; Mariotti, 2006).

This work is aimed at studying the influence of the DGS (in particular, GeoGebra) in secondary students´ mathematical competencies development, by means of a design research experience. It was implemented along two cycles at a secondary school in Almeria (Spain). In the next sections, the framework for the study of mathematical competencies, the methodology employed, the discussion of the results, and some final implications will be presented.

Mathematical Competencies

The PISA Project (OECD, 2006, 2009) has been adopted as a framework for the study of the mathematical competency. As above mentioned, this mathematical competency can be considered as the sum of various competencies. The eight competencies considered by the PISA Project are (a) Thinking and Reasoning, (b) Argumentation-Demonstration, (c) Communication, (d) Modeling, (e) Problem Posing and Solving, (g) Representation, (h) Using Symbolic, Formal and Technical Language and Operations, and (f) Use of Aids and Tools. These competencies are transversal to the different mathematical topics and can be mastered at different levels. Even though they are interrelated, this work considers seven of them separately. The competencies Representation and Using Symbolic, Formal and Technical Language and Operations were regarded as one, due to the interrelation between them, as well as to the mathematical topic developed in the experience: “Tessellations of the plane”, in which managing formulae and equations is not relevant. As for the competency Problem Posing and Solving, only problem solving was looked upon. In what follows, we briefly introduce each mathematical competence and present some evidences of the influence that DGS may have on students´ mastering them. We shall refer specifically to the geometrical domain, which constituted the focus of our study.

Thinking and Reasoning (TR)

This competency involves posing and answering questions, distinguishing between different kinds of statements (definitions, conjectures, examples, etc.), and understanding and handling the extent and limits of mathematical concepts. Traditional curricula of primary and secondary education focus on the necessity for students to learn a list of definitions and properties of shapes. Instead of memorizing properties and definitions, students should develop personally meaningful geometric concepts and ways of thinking that enable them to analyse problems (Battista, 2001). Olkun, Sinoplu, and Deryakulu (2005) suggest that the use of DGS supports and encourages students to develop and understand conceptual systems by using geometrical properties to analyse shapes and to achieve higher levels of geometric thinking, instead of memorizing a list of properties of figures.

Argumentation-Demonstration (AD)

In the last decades, the discussion over the difference between argumentation and mathematical proof has become increasingly relevant. According to Duval (1999), argumentation can be considered as a process in which the discourse is developed with the specific aim to convince someone of the truth or falseness of a particular statement. On the other hand, proof consists in a logical sequence of implications that lead to the theoretical validity of a statement. Mariotti (2006) points out that many studies which do not deny this dispute, have attempted to clarify the relationship between both processes, based on the idea of a possible continuity between them.

It is a widely held point of view that the DGS have opened new frontiers, linking informal argumentation with formal proof (Olivero & Robutti, 2001) and that they encourage exploration and demonstration because they facilitate the approach and verification of conjectures. Also Laborde, Kynigos, Hollebrands, and Straesser (2006), found that the use of DGS with high school students resulted in improved understanding of geometric concepts and supported the development of formal proofs by students.

Communication (C)

The importance of the language in the learning of Mathematics, as well as the ability to communicate mathematically, is a topic of particular relevance (Duval, 2002). In recent years, it has been emphasized the importance not only of the written discourse, but the need of an analysis of the oral discourse to understand how learning occurs. Christou, Mousoulides, Pittalis, and Pitta-Pantazi (2004) pointed out that the DGS led to greater student participation during the phase prior to the demonstration, in which students were convinced of the validity of their guesses.

Modeling (M)

The term “modeling” has been developed by several researchers and recent interpretations point at the significant role that it plays for the development of students’ mathematical meanings in simulations and problem solving, especially when DGS is used (Pierce & Stacey, 2011). In general, the modelling process with DGS, as Schumann's model (2004), are quite similar to that described by the OECD as mathematization and the principal difference is the possibility of interacting dynamically with 2D and 3D geometric objects. In this case, the dragging is especially useful for interpreting the modelling process, the result, and the mathematical solution in terms of the original problem; thus validating the whole process (Arzarello, Micheletti, Olivero & Robutti, 1998; Guven & Kosa, 2008; Olivero & Robutti, 2001; Santos-Trigo, 2008).

Problem Solving (PS)

Problem solving should be the backbone of the Mathematics curriculum. Indeed, the rest of the mathematical competencies support this one. Many studies report how the use of technological tools can help students to develop this competence. Underwood et al. (2005) and Yerushalmy (2005) point out different attributes of these tools that support students when they face with mathematical problem solving, among others: the possibility to test ideas, to receive feedback, or to manipulate objects. According to Santos-Trigo (2008), certain aspects of mathematical problem solving improve with the use of DGS. For example, they foster the need to think the problems in terms of relevant properties of the figures, and they facilitate the verification process of conjectures by considering different methods including the visual, empirical, dragging and formal proofs to support the conjecture. Also Sinclair and Yurita (2008) analyse students´ improvement in problem solving by focusing in the ways in which DGS help them to reason and demonstrate.

Representation (R)

According to Lupiáñez and Moreno (2001), traditional analytical representations in mathematics have been widely complemented and enriched by new technologies, which have added a dynamic component to the static character they usually presented. The term executable representation is included among representation systems and, according to these authors, it carries the potential to simulate cognitive actions, regardless of who the user is. Within the DGS, the executable representations are particularly relevant, as they become manipulable. That is, they allow acting directly upon them, making it possible to display certain properties of mathematical objects (Hanna, 2000) and dynamic interrelations between different mathematical concepts (Bayazit and Aksoy, 2010), being this a clear advantage over the static representations that improve the quality of teaching and learning of mathematics (Cengel and Karadag, 2010)

Another important aspect to be considered in this competence is the ability to convert between the different representations of the same mathematical object or situation, which is considered as a fundamental process for understanding (Duval, 2002). Students have difficulties when they are solving problems that require connections and relationships among different representations of an object or situation. In this sense, working with DGS can be very helpful, by providing different representations of the same object and allowing to perform a great deal of physical manipulations, which would be harder or impossible to do by hand; thus helping the exploration, examination, comparison, and verification of conjectures during problem solving activity.

Use of Aids and Tools (UAT)

It is important that students develop the ability to manage different aids and technological tools to ensure an appropriate mathematical and, at the same time, technological literacy. The existing resources must be adapted to students, and it's worthless to use a powerful tool to solve geometric problems, if it does not help the development of mathematical competence. GeoGebra user-friendliness was one of the determining factors to choose it for this research, since the intention was not to spend time training students in the use of the software, but to make a brief presentation of the program management and the various possibilities it offers and let the students to investigate by themselves the best way to take advantage of it, i.e. to encourage the free exploration of the various software tools.

For each competency, PISA describes three levels of mastery: reproduction, connections and reflection, according to the types of cognitive requirements needed to solve various mathematical problems (OECD, 2003). At the reproduction level, students must reproduce practiced material and perform routine operations. At the connections level, students are required to integrate, connect and modestly extend practiced material. Finally, at the reflection level, students need to apply advanced reasoning, argumentation, abstraction, and modeling to new contexts.

In our study, indicators were given for each competency in order to characterize the three levels of proficiency in the geometrical domain (García, 2011). As an example, we present the Argumentation-Demonstration competence indicators, which were elaborated considering the PISA levels and the work of Gutiérrez (2005):

1. **Level 1:** Reproduction. The student is able to classify geometrical objects by their attributes. He or she can do empirical demonstrations of the naive type, i.e., the verification of the geometrical property is made by using visual aids or by observing mathematical elements or properties of one example.
2. **Level 2:** Connections. The student can classify geometrical objects by their attributes and present informal arguments. He or she can empirically demonstrate geometric properties and relationships by means of a crucial example or, going further, with a generic example.
3. **Level 3:** Reflection. From various examples, the student is able to extract rules and generalizations, and to complete deductive arguments. He or she can determine inconsistencies in given arguments, and recognize differences and similarities by contrasting characteristics or properties of geometrical objects. He or she can make deductive proofs.

According to the PISA framework, it is possible to understand each level of mathematical competence in relation to the skills with which the student performs particular mathematical tasks (OECD, 2003). Since the purpose of this study was to study the influence of the DGS GeoGebra in secondary students´ mathematical competencies development, a sequence of tasks was designed in order to assess the levels at which students were performing and whether there was an evolution along the process. The skills or capacities that students could activate in the different tasks were made explicit for each task and served as the link between tasks and competencies level of development, as we shall explain in next section.

Methods

Classroom-based research which explodes the real power of technology for education requires innovative approaches that overcome the traditional class comparison (Duval, 2002). Under the emergent paradigm of Design Research (Confrey, 2006), a teaching experiment (Confrey & Lachance, 2000; Molina et al., 2011) was conducted by a research team including two researchers in mathematics education and a secondary mathematics teacher (García, 2011). It was carried out in two cycles.

The first cycle was implemented in two classes of a public school, each one of 21 students, aged 15 to 16 years old, during one month. The school provided classes equipped with one computer for every two children and their mathematics teacher was the second author of this article and a member of the research team. The students didn´t have any previous experience with the use of mathematical software. The purpose of this cycle was to serve as a pilot study for a second cycle. The main targets were: a) to test two different DGS: C.a.R. and GeoGebra, and select one of them for the next cycle; b) to design, implement and adjust a didactical sequence that provided the students with opportunities for developing mathematical competencies; and c) to design, implement and adjust observation instruments for competencies development. As a result of the first cycle:

- GeoGebra was the software selected for being freeware and for its potential for the teaching and learning of geometry, supported by recent research (Hohenwarter, Hohenwarter, Kreis & Lavicza (2008); Preiner (2008)). We are in accord with Preyner that GeoGebra has the potential to foster active and student centred learning by allowing mathematical experiments, interactive explorations, and learning by discovery.
- A sequence of 9 tasks for the content “Tessellations of the plane” was ready to be implemented in the second cycle. The reasons for choosing this content were that (a) it makes part of the curriculum for secondary mathematics in Spain, (b) it’s especially appropriate to be worked by means of DGS (Arranz, Losada, Mora & Sada, 2009; Mora, 2009), and (c) it permits to connect school mathematics with local history, through problems contextualized in the Alhambra of Granada. Students had to work on them collaboratively in pairs (sharing one computer), learning by guided discovery. The tasks were analysed by the research team in order to ensure that they gave the students the opportunity of developing mathematical competencies from reproduction to reflection levels in the geometrical domain. For doing so, each member of the team solved every task a priori as a secondary student could do, based on the observations in cycle one; specified the capacities that a student was supposed to show in order to solve the task; and linked each capacity with certain competencies at a certain level. Afterwards, there was a process of triangulation and agreement. In table 1, we illustrate this analysis for task 5 in the sequence. The rest of the tasks and their a priori analysis can be consulted at García (2011, pp. 654-658 and 661-666).

Task 5: The Vizier suggests that you tile the floor of the “Chamber of Abencerrajes” (this room is the sultan bedroom in the Alhambra of Granada) using tiles with all equal sides, in order to save money (these tiles are cheaper than those with irregular shapes). What shapes can these tiles have? Can you obtain more regular mosaic using regular tiles with more sides? Justify your answers.

Table 1. Capacities, Competencies and Levels for Task 5

Capacities	Competences	Level
C.1. He/she interprets the formulation of the real life problem in mathematical terms	TR, M, R	3
C.2. He/she expresses the strategies employed and the results obtained, orally and in writing		
C.2.1. He/she expresses him/herself orally with his/her own words	C, PS	1
C.2.2. He/she expresses him/herself orally with proper mathematical vocabulary	C, PS	2
C.2.3. He/she expresses him/herself in writing with his/her own words	C, PS	1
C.2.4. He/she expresses him/herself in writing with proper mathematical vocabulary	C, PS	2
C.3 He/she distinguishes representations of regular tessellations, drawing and using isometries	TR, M, PS, R	2
C.4. He/she creates regular tessellations by using one type of isometry	TR, M, PS, UAT	3
C.5. He/she identifies equal angles that exist at a vertex	TR, AD, R	2
C.6. He/she argues why there are only 3 regular tessellations of the plane		
C.6.1. He/she uses only visual arguments, presenting concrete examples chosen without criterion	TR, AD, M, PS	1
C.6.2. He/she obtains the condition for tessellating the plane with regular polygons (angle=divisor of 360º)	TR, AD, M, PS	3
C.6.3. He/she argues the uniqueness of the 3 regular tessellations (there aren't more divisors of 360º)	TR, AD, M, PS	3
C.6.4. He/she doesn't show any kind of argument	TR, AD, M, PS	0
C.7. He/she understands teachers´ or other students´ statements	DAY, İ	3
C.8. He/she handles the GeoGebra software		
C.8.1. He/she can draw regular polygons	DAY, AGK	2
C.8.2. He/she properly handles the different isometric tools of the program, in order to make tessellations	DAY, AGK	3

Note: TR=Thinking and Reasoning, AD=Argumentation-Demonstration, C=Communication, M=Modeling, PS=Problem Posing and Solving, R=Representation, UAT=Use of Aids and Tools

- The following observation instruments were selected for taking data in the next cycle: Observation Grids (the left column of the tables designed for all tasks, analogous to table 1, would be used as an observation grid to be filled up by the teacher-researcher and contrasted by an external observer); Diary of the class of the teacher-researcher; Students written responses to the tasks; GeoGebra files; and audio recordings (during the problem solving processes, each student used headphones with a microphone and specific software to record his or her comments and the dialogue with the partner).

The second cycle was implemented in two classes of the same public school as cycle one, along a period of two months. Each class had 23 students, aged 15 to 16 years old who didn´t have previous experience with any mathematical software. Prior to the didactical sequence with GeoGebra, a number of sessions in both classes were dedicated to familiarize the students with all the methodological aspects of the design, except for the use of the mathematical software (namely, solving contextualized problems in pairs and learning by guided discovery). Observation grids, such as the one shown in table 1, were designed for the paper and pencil tasks (PP tasks) of these previous sessions, in order to assess competencies level in students at the beginning of the GeoGebra experience. The purposes of the second cycle were:

- To study students´ level of competencies achievement during the work with DGS (GeoGebra).
- To describe the process of competencies development.
- To explore the influence of the software, of the tasks, and of the interactions with the partner and with the teacher in competencies development.

Data were gathered at two levels: the whole class (two classes) and a sample of 12 students, 6 in each class. These students were selected taking into account a combination between competencies level of achievement prior to the GeoGebra experience and attitude towards the subject, in order to have a representative sample in each class.

The data analysis was also carried out at two levels: the two whole classes and the sample. For the whole group of students, the teacher-researcher employed the indicators of the observation grids for registering and analysing the information in her diaries. These grids were filled by the teacher-researcher and by a trained expert at random sessions. The interobserver agreement obtained a Kappa index=.87. The observation grids also served to the research team for analysing students´ written solutions to the task and GeoGebra files for each couple of students, at the end of each task. The intraobserver agreement was obtained through the triangulation of these data and consensual agreement (Boykin & Nelson, 1981; Mudford, Taylor & Martin, 2009), which enabled to inform about competencies development for each class at a general level. Since observation grids permitted to record the capacities that every student showed when solving each task, and there was a correspondence of these capacities with competencies levels of achievement, by using Excel and SPSS software, a percentage of the capacities associated with each competency for each student could be obtained. Subsequently, a student was placed at a certain level if he/she had obtained at least 75% of the capacities associated to that level and had reached at least 75% in previous levels.

For the 12 students of the sample, the data collected by means of their written solutions to the GeoGebra tasks (GG Tasks), their GeoGebra files, their audio recordings, and the classroom observation grids were analysed using the software for qualitative analysis Atlas.ti v 6.0. To do so, the first step was to insert the audio recordings, the files and the written protocols of every task for each student in the sample. Then, these data were integrated in a way that the student´s work with GeoGebra would be reconstructed for each task. The next step was to codify the material, taking as codes the capacities associated to competencies at different levels. Encoding and analysing in detail each significant piece of every session, by means of Atlas.ti, permitted to understand the process of competencies development and to report on the factors responsible for this development: attributes of DGS (Santos-Trigo, 2008), interaction with the partner and/or the teacher, or the task itself. This process was reviewed and approved by an expert in the use of Atlas.ti and in analysing classroom discourse.

Results and Discussion

Since the first cycle of the experiment can be considered as a pilot study for the second one, in this paper, we report and discuss the results of this last cycle.

Levels Achieved in Competencies Development

The results for the 46 students in both classes showed that competencies development was not homogeneous, that is, the level reached by the group of students for the different competencies varied. Nevertheless, students´ global mathematical competency improved during the experience with DGS. Before the experience, only 4 of the 46 students showed a high level of competency, while the rest manifested a low-medium level. At the end of the experience, most students had reached a medium-high level and a group of 14 students manifested a high level of development in all the seven competencies. It was also observed that, even though all the students benefited from working with DGS, for those ones with a lower cognitive level prior to the experience, the software provided a more considerable boost. Table 2 shows the global mathematical competency of the 46 students before (PP tasks) and after (GG tasks) working with GeoGebra.

Table 2. Levels in Students' Competencies Development

	Global Mathematical Competency	
Level	**PP**	**GG**
0	28.3	2.2
1	34.8	17.4
2	28.3	50.0
3	8.7	30.4

Note: Sample = 46 students. Values expressed in percentages.
PP = Paper and pencil tasks, GG = GeoGebra Tasks

The results of competencies development for the 12 students of the sample were analysed in more depth. Table 3 shows the evolution for each of the seven mathematical competencies considered in the study. These results support the global appreciation that not all of them were equally developed due to the work with DGS.

Table 3. Levels of Competencies Development in Sample Students

		Level				
Competencies	**Task Type**	**0**	**1**	**2**	**2-3**	**3**
TR	PP	25.0	25.0	41.7	.0	8.3
	GG	.0	8.3	8.3	41.7	41.7
AD	PP	50.0	33.3	8.3	.0	8.3
	GG	.0	8.3	16.7	41.7	33.3
C	PP	33.3	33.3	25.0	.0	8.3
	GG	.0	16.7	25.0	25.0	33.3
M[a]	PP	16.7	41.7	33.3	8.3	
	GG	.0	8.3	0.0	91.7	
PS	PP	16.7	33.3	41.7	.0	8.3
	GG	.0	8.3	8.3	16.7	66.7
R	PP	16.7	33.3	41.7	.0	8.3
	GG	.0	8.3	8.3	.0	83.3
UAT[b]	PP					
	GG	.0	8.3	.0	8.3	83.3

Note: TR = Thinking and Reasoning, AD = Argumentation-Demonstration, C = Communication, M = Modeling, PS = Problem Posing and Solving, R = Representation, UAT = Use of Aids and Tools

[a] Tasks in the sequence required up to level 2 (Connections) of the Modeling competency for their resolution, except for task 9, which required a level 3 (Reflection) in this competency.

[b] Use of Aids and Tools couldn't be observed in the PP tasks.

The work with DGS made a significant contribution to students´ development of mathematical competencies, generally speaking. Nevertheless, its influence was more remarkable for some competencies than for others. It was outstanding in the case of Representation and Use of Aids and Tools, in which the software helped almost all of the students to make a quick progress. A Reflection level in these competencies was reached by more than 80% of them. For the competencies Modeling and Problem Posing and Solving, most of the students evolved to a Connection or to a Reflection level thanks to the use of the software.

As we shall see bellow, for the competencies Thinking and Reasoning, Argumentation-Demonstration, and Communication, in this order, GeoGebra was losing ground in helping the students to acquire adequate levels. It was very useful when students had to manifest them at a Reproduction and even Connections level, but when a greater domain was requested, many students needed external help. Nevertheless, DGS enabled students, in general, to give a step forward from their previous situation. That is, those ones who didn't show evidences of these competencies in the preparatory sessions, prior to the work with GeoGebra, reached a Reproduction level in solving problems with the software, those situated at the Reproduction level evolved into a Connection level and those who already evidenced to be at the Connection level in these competencies (a very small number of students) progressed to a Reflection level of performance. It´s interesting to point out that, even though most of the students´ development followed this pattern, for some of them, the effect of working with DGS was surprising. They evolved from a null or very low level of performance to a Connection level or to a Reflection level. Table 4 shows the level of competency reached by the students in the sample before and after the experience with GeoGebra for Thinking and Reasoning, Argumentation-Demonstration and Communication.

Table 4. Levels of competencies Thinking and Reasoning, Argumentation-Demonstration, and Communication in sample students

		Student											
Competencies	**Task Type**	**S1**	**S2**	**S3**	**S4**	**S5**	**S6**	**S7**	**S8**	**S9**	**S10**	**S11**	**S12**
TR	PP	2	2	-*	3	1	2	-*	2	2	2	-*	2
	GG	2-3	2-3	3	3	2-3	2-3	2	3	2-3	3	1	3
AD	PP	2	-*	-*	3	-*	1	-*	1	-*	2	-*	1
	GG	2-3	2	3	3	2-3	2-3	2	3	2-3	2-3	1	2-3
C	PP	2	1	-*	3	1	1	-*	2	-*	2	-*	1
	GG	2	2	3	3	2	2-3	1	3	2-3	2-3	1	3

Note: Values express competencies level.
AD=Argumentation-Demonstration, C=Communication,
PP =paper and pencil tasks, GG=GeoGebra Tasks
* no signs of competency showed.

Progression in Competencies Development

The data analysis of the sample students showed that the development of certain competencies affected the development of other competencies. Namely, an adequate level of Use of Aids and Tools contributed to the improvement in the competency of Representation. This, in turn, fostered the progress in Thinking and Reasoning, which led to an evolution in the development of the competency Argumentation-Demonstration.

At the beginning of the process, GeoGebra turned out to be very intuitive and easy to use for students, who could reach adequate levels of proficiency with almost no previous instruction. This condition allowed them to make accurate graphic representations with less effort and in shorter time than with paper and pencil, providing extra time to think. The possibility of manipulating representations made possible to test, by trial and error, which mathematical properties remained invariant for each construction, fostering the ability to reason in terms of mathematical properties. On the other hand, thanks to the interactivity of GeoGebra, students could immediately check the appropriateness of their actions, which, in time, led them to abandon their former tendency to act at random. Progressively, their mental processes were redirected into planning goals and looking for strategies in order to reach them. This, together with the arising capacity to make complex reasoning, motivated students to look for arguments and proofs, and had an important effect in increasing the quality of the explanations and demonstrations that they provided.

In order to illustrate how the use of the properties of GeoGebra influenced the progression in Argumentation-Demonstration and Communication competencies, we show part of an episode in which a couple of students had been struggling with the explanation of the reason why it´s possible to tessellate the plane with every quadrilateral. The teacher had tried to help them along the process and, finally, she suggested to them the employ of the "dragging" tool, in order to see what happened at the vertex of a mosaic constructed by the students with a non-particular example of quadrilateral. We reproduce the figure of the original mosaic and its variation, together with the dialog that took place between the students after using the "dragging".

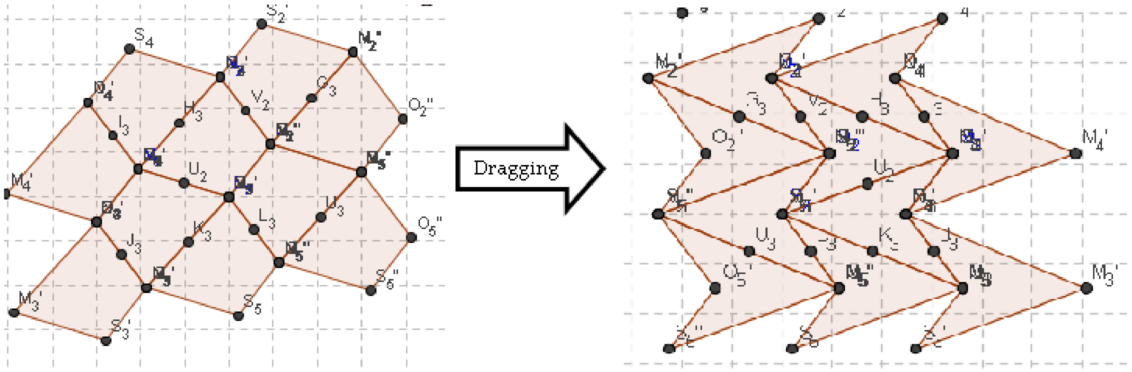

Figure 1. Original mosaic and mosaic transformed by means of dragging

S1- The angles of quadrilaterals measure 360, and ...

S2- Yes, yes, yes, the angles of quadrilaterals add up to

S1- 360 º

S2- 360, and what else?

S1- So at each vertex, having four squa..., being the four different corners of each quadrilateral, they add up to 360 º. Well at each vertex, isn`t it? Because, look at this vertex: there is the corner of the quadrilateral, then the other corner, then the third corner of the quadrilateral, and then the forth one.

S2- Ah! Because the four corners of the quadrilateral meet

S1- Sure, at each vertex the four corners of the quadrilateral meet.

S2- [writes on the task sheet] Because the angles en each vertex add up to 360º. The angles of quadrilaterals add up to 360° and, at each vertex, the four corners come together.

Factors Influencing Competencies Development

The analysis made with Atlas.ti v 6.0 allowed to inform, not only on the competencies development for the sample students, but also about which factors were more influential for this development: DGS (GeoGebra), interaction with the partner, interaction with the teacher, and/or the task. In Table 3, we can observe that most students reached a high level of competency for Representation and Use of Aids and Tools, and a medium-high level of proficiency for Modeling and Problem Solving. In these competencies, progress was evident since the beginning of the experience, quite homogeneous and GeoGebra was identified as the main factor for it. Its influence was remarkable in the case of solving open-ended problems, since previous to their work with the software, students in general had shown a great difficulty in facing them. In this sense, observations were made by the teacher-researcher after the experience with GeoGebra, when the students had to face open real world problems without the help of the program. Interestingly enough, she could detect that the experience had made the students aware of the importance of keeping employing their mathematical competencies in order to succeed in their activity. For example, students felt the need to plan their actions according to their understanding of the problem, and also intended to argue and justify their responses. These needs had rarely aroused prior to the experience. That is, the experience with GeoGebra seemed to have provoked a significant shift in students´ approach to problem solving. Unfortunately, without the support provided by the software, most of the students were limited by their previous cognitive deficiencies. Without the possibility of quickly generating and manipulating accurate examples from which to make conjectures and without the possibility to test them, their capacity to devise strategies and solve problems soon decreased. This was more obvious in the case of students with prior lower cognitive levels.

For competencies Thinking and Reasoning, Argumentation-Demonstration and Communication, the evolution was heterogeneous, more gradual and many students needed support from the teacher and/or from their partners in order to progress. Table 5 shows the influence of each of these factors in the development of the competencies Representation and Argumentation-Demonstration for the students in the sample.

Tablo 5. Factors Influence on Representation and Argumentation-Demonstration Competencies

		Factor			
Competencies	**Level**	**GG**	**IP**	**IT**	**T**
R					
	1	100.0	.0	.0	.0
	2	79.1	13.2	7.7	.0
	3	65.7	17.1	10.0	7.2
AD					
	1	90.0	10.0	.0	.0
	2	51.3	27.8	20.9	.0
	3	19.5	21.2	47.0	12.3

Note: Values expressed in percentages.
AD=Argumentation-Demonstration, GG=GeoGebra, IP=Interaction with the partner,
IT=Interaction with the teacher, T=task

So, even though DGS was the most important factor in general for fostering competencies development in students, its influence depended upon the different competencies, as well as upon individual students. As the cognitive demands of the tasks increased, social interaction gained relevance for the evolution of the competencies Thinking and Reasoning, Argumentation-Demonstration, and Communication, in which students showed more difficulties. For most of them, the interaction with the teacher was necessary in order to advance. She gave them suggestions or help, aimed at making them devise their own strategies, properly argue, and communicate. For some couples of students, their collaborative interaction became decisive in order to progress, while the interaction with the teacher was relegated to a second plane. This was the case when one of the students was more competent than the other, but the second one was receptive and capable enough to benefit from the help of the former. In any case, the interaction of each student with his or her partner was not beneficial in many cases, but it was not detrimental either (a detailed study of the interplay between the influence of the software and of the social interaction in this study can be consulted at García, Romero & Gómez-Chacón, 2014).

Conclusions

In our study, we have addressed two current needs in mathematics education: introducing the use of new technologies in real classrooms, in particular DGS, and assessing its effects in terms of its influence in students´ mathematical competency. Instructional and assessment materials are available for other teachers and researchers, in order to contrast the results of this study (García, 2011). In particular, observation grids designed for the tasks link observable actions performed by students to mathematical competencies development, which takes place along a period of time. This resource has been used in regular classrooms and can be easily adapted to other mathematical topics and tasks. Therefore, it can help teachers who currently need to assess competencies development in their students.

The results obtained in this study are in line with those of other works dealing with the influence of DGS in mathematical competencies development. We can mention the need to reason in terms of geometrical properties (Hanna, 2000), the software potential to verify conjectures (Christou, Mousoulides, Pittalis, & Pitta-Pantazi, 2004), and the advantages found in using the software for developing competency in problem solving by Underwood et al. (2005) and Yerushalmy (2005).

Our work also gave information on other competencies, and provided a comparison among degrees of development. DGS turned out to be a powerful resource for the development of those competencies which were more related to visualization processes. That is, Use of Aids and Tools, together with Representation were the two competencies that got better results from the beginning for almost all the students. The software, by means of its attributes and properties, made an important contribution to the development of those competencies related to mathematization, namely, Modeling and Problem Solving, in which most of the students significantly improved their levels of performance and reached a good degree. Students with prior lower levels of mathematical competency deserve special attention, since their progress due to the use of DGS was more evident and they showed more dependency on the use of DGS attributes in order to maintain their new gained skills. It would be interesting to explore ways to keep up with this progress and to determine to which extent the use of this kind of technology is indispensable for this purpose.

On the other hand, the effect of DGS decreased for those competencies linked to reasoning and discursive processes, namely, Thinking and Reasoning, Argumentation-Demonstration, and Communication. Students´ progress was slower and not all of them reached appropriate levels. Nevertheless, the software helped students, in general, to give a step forward from their previous situation. There were, however, some exceptions: for certain students the effect of working with the software was surprising, since they went from zero or very low initial levels to medium and high levels in the above mentioned competencies. In our opinion, this is a fact that deserves attention, since it shows that for some students, mathematical capabilities can remain unseen, and therefore undeveloped, for their teachers and for themselves. They could find in new technologies, specifically in working with mathematical software, a way out for their latent mathematical ability.

In relation to the factors that contributed to the evolution of the competencies studied, the data analysis revealed that social interaction had a role to play. Even though DGS GeoGebra was the most important factor for fostering progress in the seven competencies, up to a certain level for each, as the demands of mathematical activity dealing with logical reasoning, argumentation and use of mathematical language increased, the importance of other factors became evident: namely, interaction with the teacher and/or with mates, in this order (in the particular classroom dynamics for this experience). Therefore, even though DGS can indeed benefit the teaching-learning process, they cannot take the place of the teacher and of social interaction in the mathematics classroom. More research is needed on how teachers can make a proper use of the of DGS attributes in order to improve instrumentation and visualization processes in their students. There is also a need to explore how teachers can extend these benefits, well documented in the literature (Coutat, 2006; Coutat & Richard, 2011), and take advantage on them in order to develop discursive processes in their students, since these processes lie at the root of higher order mathematical thinking.

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