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Original Article

First-Grade Students' Strategy Choices in Addition Word Problems *

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Abstract

The aim of this study was to understand the factors that play a role in the first-grade students' strategy choices while solving addition word problems. Six students selected from two first-grade classrooms were the focus participants of the study. Data were collected through five clinical interviews conducted throughout the academic year 2018-2019. The interviews were analyzed based on Baroody and Ginsburg's framework (1986), including three factors: (i) semantic structure of problem, (ii) cognitive economy, and (iii) problem size. We found that the first-grade students who did not receive explicit instruction on a particular strategy could develop several strategies at different levels of sophistication. Regarding the factors influencing students' strategy choices, our findings confirmed Baroody and Ginsburg's (1986) framework and extended it with three additional factors: (i) the semantic structure of the mathematical operation and the order of the numbers stated in the problem, (ii) a flexible and conceptual use of number combination families for the cognitive economy, and (iii) the numbers involved in the problems. Hence, strategy choices and factors influencing strategy choices were discussed for educational implications.

Keywords

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Introduction

"Learning arithmetic as problem-solving" is an important research area in early mathematics education (Verschaffel, Greer, & DeCorte, 2007, p. 559) because a solid understanding of numbers and operations enhances not only present but also future learning of mathematics. The importance of developing strategies for arithmetic operations relies on this progressive support for the learning of mathematics. More specifically, the research has shown that students who developed a repertoire of mental arithmetic strategies could extend their understanding of different problem situations in later grades (Bailey, Littlefield, & Geary, 2012; Verschaffel et al., 2007).

* Erdinç Çakıroğlu, one of the authors of this study, sadly passed away during the printing process of the manuscript after the evaluation and all corrections were completed. The article is dedicated to his memory by the first and second authors.

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Strategy development took mathematics education researchers' interest because arithmetic strategies involve critical cognitive operations such as counting, composition, and decomposition, provide "cognitive advantage" (Clements & Sarama, 2007, p.474), and "reduce the burden on rote memory" (Jordan, Kaplan, Locuniak, & Ramineni, 2007, p. 44). Hence, engaging in different strategies constitutes a foundation for analytical reasoning and needs to be at the center of early mathematics, and developing ways of enhancing students' strategy repertoire at early ages took researchers' interest (Sunde & Sunde, 2019). Therefore, developing strategies for four operations have been addressed in many countries' mathematics education curricula (Ministry of National Education [MoNE], 2018; NCTM, 2000).

Despite this emphasis in elementary school mathematics curriculum, research particularly conducted in Turkiye showed that students tended to use standard algorithm (Kayhan-Altay, 2023) and that students' mental computation strategies were limited (Duran, Doruk, & Kaplan, 2016; Güç & Hacısalihoğlu, 2016). Furthermore, Güç and Hacısalihoğlu (2016) claimed that students tend to use strategies that they are familiar with, although those strategies do not necessarily lower the cognitive load. These studies implied the need for an understanding of why students favored some strategies over others. In response to this need, we aimed to understand first-grade students' various strategies developed while solving addition word problems and what factors might influence their strategy choices. We particularly addressed the following research questions:

1. What are the strategies that students use in solving addition word problems?
2. What factors influenced the first-grade students' strategy choices in addition word problems, and how?

To investigate these research questions, we examined the case of addition-focused interviews of six first-grade students at a public elementary school in Ankara. We present the results of the analysis of student interviews in light of the literature on students' mental strategies and the theoretical framework of factors influencing strategy choices, which are explained in the next section.

Theoretical Framework

We framed our study around students' strategies on arithmetic problems and factors influencing strategy choices of students.

Students' Strategies for Arithmetic Problems

Mathematics education and childhood education researchers extensively investigated students' development of number sense and operations (Fuson, 2003; NCTM, 2000). The research studies in this area, in fact, present two different perspectives; while some research support strategy instruction aiming to teach particular strategies to students (e.g., Baroody, Purpura, Eiland, & Reid, 2015; Sunde & Sunde, 2019), others suggest organizing the learning resources and cognitive building blocks to support students' development of their strategies (e.g., Chu, Rouder, & Geary, 2018; Mulligan, 2004; Schiffman & Laski, 2018). For instance, while Sunde's work involved investigating the instructional practices of changing students' strategies, Mulligan (2004) suggests fostering children's numbers sense in the early school years to develop numerical strategies besides teaching traditional algorithmic procedures. On the other hand, both perspectives confirm that strategies play an important role in problem-solving (Pongsakdi et al., 2020; Sievert, van den Ham, Niedermeyer, & Heinze, 2019; Verschaffel, Schukajlow, Star, & Van Dooren, 2020) even in later grades (Bailey et al., 2012; Verschaffel et al., 2007).

Researchers suggested that part-part-whole knowledge of numbers is central to developing strategies for solving arithmetic word problems (Carpenter, Fennema, & Franke, 1996; Ding & Auxter, 2017; Verschaffel et al., 2007). In fact, there is a complex and bi-directional relationship between the two. Not only part-part-whole understanding leads to the development of the invented solution strategies, but also the invented strategies also contribute to students' understanding of part-part-whole relationships. Furthermore, the part-part-whole relationship sets a foundation for further mathematical relations, such as inverse relations, and provides cognitive flexibility (Chu et al., 2018; Ding & Auxter, 2017). Besides, based on a prolonged research study, Carpenter, Moser, and Bebout (1988) concluded that invented strategies in addition and subtraction intensely interact with the underlying meaning of number concepts.

Recent studies also articulated similar findings both in the international and national arena. For example, Schiffman and Laski (2018) found that students could develop strategies based on composition and decomposition, which help students understand the numbers conceptually. Similarly, in another study, Chu et al. (2018) found that students could develop advanced strategies based on part-part-whole relations among numbers (such as number families), which helped them develop the cardinal value meaning of numbers. Likewise, Duran et al. (2016) found that not elementary but middle school students' most frequently used strategy was decomposition and composition. On the other hand, Güç and Hacısalihoğlu (2016) encountered decomposition less frequently in middle school students' mental addition work. These conflicting findings indicate a need for further investigation of students' strategy choices, which we aim to do in this study by exploring first-grade students' strategy choices in addition word problems.

Research also showed us that students' invented strategies on addition are strongly related to counting skills because as students count, they develop cardinal meanings linked to addition and subtraction (Bailey et al., 2012; Duran et al., 2016; Zur & Gelman, 2004). Guerrero and Palomaa's study (2012) confirmed that students started with counting strategies but used a variety of strategies depending on the numerical values of the addends and the sum in the problems. The most basic counting strategies are (1) counting-all and (2) counting-on (Clements, Sarama, Baroody, & Joswick, 2020; Guerrero & Palomaa, 2012; Kayhan-Altay, 2023). In the counting-all strategy, for instance, in solving 3+5, students start counting from 1 to 8. In the counting-on strategy, students keep one of the numbers in mind and count starting from that number (i.e., counting on 3 up to 8). A more adaptive version of the counting-on strategy was the counting-on-from-larger strategy, in which students count on 5 instead of 3 as six, seven, and eight because there were fewer numbers to count on from the larger number. These counting strategies indicated that students developed cardinal meanings for the "counted quantity" and successfully used this reasoning for solving addition and subtraction problems (Fuson, 1992; Sunde & Sunde, 2019).

Besides counting, students often used pairs of numbers, known as "number combination families," such as pairs of numbers making 10. For instance, break-apart-to-make-ten and incrementing tens and/or ones were the two strategies that evolved from known pairs of 10 (Clements et al., 2020). Using number pairs of 10 is one of the basic and crucial strategies for two reasons. First, considering 10 as a reference embraces various ways of using 10. Second, composing or decomposing to make 10 requires a part-part-whole relation that is at the center of many strategies. Known combinations of numbers other than 10, called combination families, were also used in inventive strategies (Baroody et al., 2015; Fuson, 1992). Specifically, the add-1 and the double combination families, such as thinking 6 as pairs of 5 and 1 or 3 and 3, constitute a base for other efficient strategies (Guerrero & Palomaa, 2012; Baroody et al., 2015). It is important to note that the number of combination families is based on relational thinking, particularly part-part-whole relations (Chu et al., 2018; Kayhan-Altay, 2023). Such relational understanding is pivotal in developing effective strategies that build upon conceptual understanding, thereby fostering problem-solving skills and modeling competencies (Schiffman & Laski, 2018; Verschaffel et al., 2020). Hence, strategy development plays a crucial role in facilitating efficient learning experiences that contribute to the advancement of students' mathematical thinking as they progress toward higher grades and develop more abstract mathematical knowledge. Therefore, recent studies focusing on arithmetic strategies emphasized developing flexibility and adaptivity in strategy choices (Russo & Hopkins, 2018; Sunde & Sunde, 2019). By adapting new strategies based on existing ones and flexibly using the more efficient strategies for specific problems-solving contexts, students can build a so-called "strategy repertoire." The absence of such a repertoire may hinder students from developing more sophisticated strategies, such as *subtraction as addition* (Verschaffel, De Smedt, Van Der Auwera, & Torbeyns, 2021).

Considering the importance of strategy repertoire, researchers (e.g., Carpenter & Fennema, 1992; Carpenter et al., 1988; Clements et al., 2020) investigated types of word problems that influence students' strategies in addition and subtraction problems. Similarly, in the most recent study, Kayhan-Altay (2023) investigated 2nd, 3rd, and 4th grade students' strategy performances using different types of problems – result-unknown problems, compensation problems and covariation problems – and found that students at all grades performed better in the result-unknown type of the addition problems. Furthermore, this study showed that elementary school students, excluding first graders, tended to use standard algorithm (Kayhan-Altay, 2023). Similar observations were also reported in other studies such that students' mental computation strategies were limited (Duran et al., 2016; Güç & Hacısalihoğlu, 2016). In this regard, Güç and Hacısalihoğlu (2016) claimed that students tend to use familiar strategies, and since standard algorithm is the most frequently used method in the instruction and the textbooks, students' tendency of using standard algorithm could have been affected by those.

The potential for strategy development in arithmetic problem-solving may also be influenced not only by the nature of the problems posed but also by the availability of learning resources such as number lines (Schiffman & Laski, 2018) and textbooks (Sievert et al., 2019). In this regard, studies investigating primary school teachers' competencies highlighted that the teachers also considered the importance of the manipulatives for the development of strategies, such as a hundred-chart for counting strategies (Kalaycıoğlu Akis & Şahin, 2023; Üstündağ & Özçakır Sümen, 2023). Regarding the textbooks as a source for a range of strategies, Bütüner (2020) compared Turkish and Singaporean 3rd and 4th grade mathematics textbooks and found that Turkish textbooks included relatively limited opportunities for strategy development, compared to Singaporean textbooks.

Besides, Guerrero and Palomaa (2012) found that addition involving multi-addends and multi-digit numbers may promote more varied strategies than single-digit addition problems. In other words, the size of the numbers also influences the range of strategies students might develop (Guerrero & Palomaa, 2012; Verschaffel et al., 2007). Kayhan-Altay's (2023) study also confirmed that a particular selection of the numbers in compensation and covariation problems increased the cognitive load of the problems, and 2nd, 3rd, and 4th grade students had difficulty developing a strategy for those problems. On the other hand, Pongsakdi et al.'s study (2020) showed another perspective that numerical factors did not appear as deterministic factors for the perceived difficulty of word problems.

As mentioned earlier, strategy-focused instruction produced various results (c.f., Baroody et al., 2015; Chu et al., 2018; Mulligan, 2004; Schiffman & Laski, 2018; Sunde & Sunde, 2019); however, the studies argued that even though the teacher did not provide an instruction aiming to teach a particular strategy, their instructional practices and use of resources influence students' strategy repertoire (Schiffman & Laski, 2018; Sievert et al. 2019; Sunde & Sunde, 2019). In this regard, studies investigated Turkish elementary school students' strategies found that third graders showed a better performance compared to other grade levels, and researchers explained this difference, although not necessarily statistically significant, by curriculum (Bacakoğlu & Tertemiz, 2022; Kayhan-Altay, 2023). More specifically, since the third-grade mathematics curriculum included mental addition strategies such as rounding, number pairs (or number families), counting on, and decomposition (MoNE, 2018), they connected third-graders' performances with strategy instruction.

In addition, Baroody et al. (2015) argued that although some level of guidance and particular instruction might influence students' development of particular strategies, students could still develop a variety of strategies without explicit instruction on strategies when an appropriate learning environment was provided (Torbeyns, Verschaffel, & Ghesquière, 2005). Children even had a large range of strategies like counting, grouping, and partitioning before they encountered formal education (Wright, 1998; Wright, Mulligan, & Gould, 2000). Similarly, in Kayhan-Altay's (2023) study, even though rare, second-grade students could use part-part-whole relation for developing a strategy to solve addition problems without having a particular instruction. Yet, teachers are the key agents who could elicit those strategies that students brought to the classroom, and therefore, improving teachers' knowledge and skills in identifying cognitive affordances and constraints of the strategies and using multiple strategies in teaching mathematics need to be considered in this area of research (Durkin, Star, & Rittle-Johnson, 2017). Hence, by investigating first-grade students' strategy choices in addition word problems, the current study might contribute to teachers' knowledge about promoting students' strategy repertoire.

A Theoretical Framework on Factors Influencing Strategy Choices

Baroody and Ginsburg (1986) proposed a framework for strategy choices based on three influential factors: (i) the semantic structure of the problem, (ii) cognitive economy, and (iii) problem size.

The first factor, *the semantic structure of the problem*, is related to the type of the problem, such as whether the problem involves a combined and changed situation (Cowan, 2003). These situations are critical in the sense that they describe the cognitive operation that students would go through to solve the problem. For instance, the combine problem involves combining two sets of items either physically or mentally (e.g., "Peter has 3 apples, and Anne has 7 apples. How many apples do they have altogether?" (Baroody & Ginsburg, 1986, p. 86)) and signifies a binary conception of addition. On the other hand, a change problem (e.g., "Pete has 3 apples. Ann gave him 7 more. How many does Pete have now?" (Baroody & Ginsburg, 1986, p. 86)) describes a change in the cardinality of one set and therefore implies a unary conception of addition. The researchers argued that students' tendency to use the counting on-from-the larger strategy in combining problems and counting on-from-the-first strategy on the change problems might be related to the semantic structures of the problems.

The second factor, *cognitive economy*, is associated with the cognitive demand of the problem and suggests that students have a tendency to prefer strategies involving less demand on working memory. In this regard, students may invent new strategies that would "reduce the load on working memory" (Baroody & Ginsburg, 1986, p. 86). For the addition problems involving a smaller number as the first addend and a larger number as the second addend, counting on-from-the larger strategy would require lower counting efforts than counting on-from-the-first strategy does, and therefore appear as a cognitively more economic strategy choice (Cowan, 2003).

The last factor, *problem size*, also indicates cognitive economy but involves some degree of evaluation of the effectiveness of a particular strategy in problems involving larger numbers. For instance, for the problem of 4+22, students' strategy choice would be counting on-from-the larger since counting on 22 four more would involve less effort than counting on 4 twenty-two more. Thus, students evaluated the cognitive economy in relation to the problem size and determined that counting on-from-the larger strategy would be more efficient, particularly when the problem involves a larger number as one of the addends (Baroody & Ginsburg, 1986). Although this factor may indicate the careful selection of the numbers involved in the problem, it is specifically about using larger numbers to promote certain strategies. For instance, for the problem 4+8, the numbers are selected considering that one number is close to 10, and so may lead to the "making a ten" strategy. However, the third factor, problem size, does not involve this case because both numbers in problem 4+8 are one-digit numbers. Therefore, in the *problem size* factor, the researchers pointed out the purposeful use of at least one larger number in the addition problem.

The above-mentioned factors did not involve an explicit teaching of strategies; rather, they are drawn from the nature of the problems. In this study, we put this issue under the spotlight and examined how and to what extent those factors affected students' strategy choices in solving three different addition problem types, keeping in mind the other factors that could also be effective in students' strategy choices.

Methods

The present research constitutes a subset of a larger study that employs a design-based research approach, encompassing a two-year iterative investigation conducted in a public elementary school in Ankara, Turkey, during the 2017-2018 and 2018-2019 academic years (see Çakıroğlu, Işıksal-Bostan, & Sevinç, 2019 for details of the larger project). The school was a regular elementary public school, which was relatively small. More specifically, there were two classrooms at each of the four grade levels. Therefore, we included both first-grade classrooms in our study. Hence, in each year of the larger study, we worked with two first-grade classrooms (each with 18-22 students), and explored hypothetical learning trajectory in iterative cycles, involving a continuous process of refining design, collecting and analyzing data, and incorporating feedback and insights obtained from earlier stages of the study. As part of our methodology, we carried out a series of interviews with selected students throughout the larger design-based research.

The study that we reported here focuses on the case of interviews with selected students in the second year of the larger study. Therefore, we do not present the entire design-based research process rather a case of interviews focused on addition word problems with selected students, which were analyzed through content analysis.

Participants

In this study, we focused on six students selected from two first-grade classrooms. To select the case students, initially, we discussed with classroom teachers to gather their insights and observations on students who exhibited talkative behavior in the classroom. In addition to those teacher observations, researchers also relied on their observations and the results of the Number Knowledge Test (Okamoto & Case, 1996) administered at the beginning of the school year. The addition interviews were held toward the end of the Fall semester, and the research team were present in the classroom every day to monitor the learning trajectory that was developed in the larger design-based research. Thus, the researchers' field experience notes helped us to select the case students for the interviews.

Through an analysis of classroom observations and test results, students who demonstrated an average level of number knowledge performance were identified. Finally, from this pool of students, six participant students were randomly selected for inclusion in the study. The Number Knowledge Test involved questions such as identifying a particular quantity, recognizing a stated number, and identifying 1 or 2 more/less than a particular number without using any visual or concrete manipulatives. Based on the students' performances, the test included an evaluation rubric that identified students' developmental age of number knowledge (Okamoto & Case, 1996). According to the test results, all students, except Tilbe, were at the developmental age of 6–7 years, indicating that they could perform simple number tasks without relying on concrete manipulatives. For instance, they could identify single-digit numbers and relate them with one or two numbers apart. On the other hand, Tilbe was at the developmental age of 5–6 years and relied mostly on manipulatives to find the number of items in a given set of items. Hence, students' initial number knowledge did not involve relational thinking of numbers and addition operation at the beginning of the semester. Table 1 below presents the demographics of the students we followed throughout the year via a series of interviews.

Table 1. Information about focus students

Student Names (Pseudonyms)	Gender	Duration of Kindergarten Education
Fuat	Boy	4 years
Ayaz	Boy	2 years
Zehra	Girl	3 years
Metin	Boy	2 years
Irmak	Girl	2 years
Tilbe	Girl	3 years

Although we did not consider students' kindergarten education period neither in our selection of the case students nor in interpreting the findings, the above demographics provide an overview for the case students. For ethical considerations, we received Institutional Ethics Committee approval. Based on the ethical regulations, the parental consent forms were received before the data collection process; pseudonyms were used to present the findings.

Instructional Intervention

Before the study began, we worked with classroom teachers to develop an understanding of number sense and invented strategies based on the literature. In the project's first year, we focused more on the data-driven revisions of the instructional intervention. In the second year, we directed our attention to students' cognitive thought processes. The findings of the current study are driven from part of the data obtained in the second year of the larger study.

As we worked with the classroom teachers, we aimed to develop an insight into connecting number sense and operations by enabling students to fluently use their reasoning, conceptual understanding, and procedural skills to solve addition and subtraction problems flexibly. We put a particular emphasis on number combination families through representations (see Figure 1) in teaching numbers and operations meaningfully. Before students learned the addition operation, number combination families were conceptually introduced as "number bonds" with the help of manipulatives and visual materials. While learning numbers, number bonds were used in various ways to represent groups of quantities as number combinations without making any formal reference to the addition operation. In quantification tasks, students were frequently given two groups of objects and asked to fill in the number combination families as in Figures 1a and 1b. In some cases, students were given two groups of objects, one of which was hidden, and given the sum, they were asked to infer the number of hidden objects (Figure 1c). When addition and subtraction operations were formally introduced, they were linked to the number bonds structure given in Figure 1, and both horizontal and vertical notations were used to represent the same addition or subtraction problem.

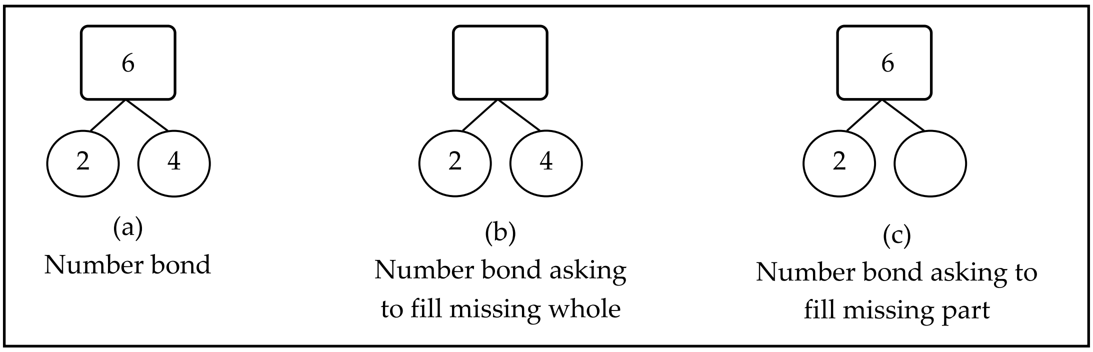

Figure 1. Sample number combination families used in the intervention

Students' thinking on number combinations was also cultivated in regular subitizing activities (Clements, 1999), in which students were asked to recognize a small group of objects without counting in a short period of time. In this way, students' thinking towards *breaking objects into two groups, subitizing the parts, and telling the total number of objects* were facilitated. This way of subitizing, known as conceptual subitizing (Clements & Sarama, 2014), was based on part-part-whole reasoning.

While working on addition word problems, students were not instructed to use specific solution strategies. They were consistently encouraged to invent strategies and share their invented strategies with the class. In most cases, the teachers encouraged students to find alternative ways to solve the problem and share it with the class. Therefore, the intervention implemented by the classroom teachers aimed to promote the strategical competencies of students in arithmetic word problems through invent-and-share routines in the classroom.

Data Collection Procedure

As part of the larger study, we conducted five clinical interviews with these six students throughout the year. In this study, we mainly focused on the interview, in which three addition word problems were posed to students in the following order:

1. ***Sum Unknown Problem:*** There are 12 trees in our school garden. When we plant 9 more trees, how many trees will there be altogether?
2. ***Second Addend Unknown Problem:*** Sude has 6 toys. When her mom bought her some more, she had 13 toys. How many toys did her mom buy?
3. ***First Addend Unknown Problem:*** There were some students in the class. 12 more students came, and now there are 19 students altogether. How many students were there in the class at the beginning?

During the interviews, we provided students with materials such as interlocking cubes, ten-frame cards, and number bond diagrams (as shown in Figure 1) and let them know that they may use any of these materials as they needed. Furthermore, as the interviewers asked these questions, they often probed students' thinking by asking, "Can you explain to me how you found the answer?" and "Can you solve it in another way?" We took both audio and video records of the interviews and scanned all the written work of the students. While video recording the interview, we placed the camera to capture students' writings or work with materials on the table.

Data Analysis Procedure

We transcribed the audio and video data and transferred all data into a qualitative data analysis software, MAXQDA 2020 (VERBI Software, 2019), and synchronized the transcripts and video/audio interview data. The data analysis presented in this study involved content analysis (Miles, Huberman, & Saldana, 2014), which coincided with the retrospective analysis of the larger design-based research. Since the focus of this study is neither design conjectures nor learning trajectory, we presented only the content analysis of the case of addition word problems solved individually by selected first grade students in an interview setting.

Our analysis involved three phases: open coding, code categorization, and code relations. In the first phase of analysis, we carried out open coding to identify the cognitive actions that students operated to solve the addition word problems. In the second phase, we went through the entire data again and categorized these cognitive actions to name the strategies that students developed. For instance, for the problem 12+9=?, one student stated: "I took out 2 from 12 and then added 9 [to 10]. I added 1 more and 1 more. [that makes] 21." In the first phase, we named three actions: decomposing the number into ten and ones, combining ten and ones, and adding one by one. Then in the second phase, for the combination of these cognitive actions, we labeled three strategies: *making (multiples of) ten*, *using ten to compose a number*, and *dealing with larger ones.* In addition, we created maps of codes to visualize the relations of cognitive actions, which helped us to confirm the strategies. Hence, not only categorizing the cognitive actions through going over the raw data and open codes, but also through visual methods, we determined six strategies: (1) Counting on, (2) Making (multiples of) ten, (3) Using ten to compose a number, (4) Dealing with larger ones, (5) Ignoring tens and using number bonds, and (6) Doubling and add one (see Table 2 for demonstration of each strategy).

The last phase of data analysis included a series of cross-comparative analysis. First, we articulated the factors influencing students' strategies by examining the pattern of students' strategy choices. Thus, we compared the strategy codes across students. Second, we explored the range of strategies across types of addition problems, which allowed us to determine the first factor of strategy choice, the semantic structure of the problem. Third, we compared the strategies categorized by problem type across the three factors of the strategy choice framework. For instance, in the example above, students' cognitive actions aimed to lower the cognitive demand by slitting up the number into ten and ones and combining first the larger ones to ten, therefore indicated the factor of *cognitive demand*.

Triangulation was employed to enhance the study's credibility. We used multiple data sources (i.e., students' written work, video/audio data, and interview transcriptions) to learn more about the students' strategies and support the findings. Furthermore, two of the researchers coded the data together in the first two phases. Therefore, the disagreements about cognitive actions and strategy categorizations were resolved during the data coding. All three researchers discussed the results of the third phase of the analysis. Since this phase included the categorization of the factors based on the strategies of students, we resolved the disagreements by turning back to the framework occasionally. To reach full agreement, first two authors met 1-2 hours every week for 3-4 months in the first two phases of data analysis, and all three researchers met to agree on the factors identified for each student's strategy choice for each problem. Hence, the comparison of multiple researchers' perspectives and comparison with theory provided ways of triangulation for trustworthiness (Lincoln & Guba, 1985).

Findings

In this section, we first provide an overview of students' strategy choices based on our exploration across three types of addition problems (i.e., sum unknown, first addend unknown, second addend unknown). Then, we present our accounts on the factors influencing our first-grade students' strategy choices in addition word problems.

Overview of Students' Strategy Choices

Our analysis of students' solutions revealed six strategies that were mainly based on composing and decomposing parts of a whole, parallel to the ones mentioned in the literature (e.g., Clements et al., 2020; Fuson, 2003; Verschaffel et al., 2007). In Table 2, we summarized the strategies and included researcher-generated mathematical notations based on the students' explanation of each strategy, which are explained in detail in the next section.

Table 2. Summary of the strategies and researcher-generated mathematical notations

Strategies	**Researcher-generated mathematical notation based on students' work**	**Students used the strategy**
Counting On	*First Addend Unknown Problem:* __ + 12 = 19 12 → 19 (13,14,15,16,17,18,19) *Second Addend Unknown Problem:* 6 + __ = 13 6→13 (7,8,9,10,11,12,13); 6 + 7 = 13	Irmak Tilbe Ayaz Zehra
Making (multiples of) Ten	*Sum Unknown Problem:* 12 + 9 = __ 12+9 = (11+1) + 9 = (11+9) + 1 = 20+1 = 21 *Second Addend Unknown Problem:* 6 + __ = 13 (6 + 4) + 3 = 6 + (4+3) = 13 [4 + 3 = 7]	Metin Fuat Ayaz Zehra Irmak Tilbe
Using Ten to Compose a Number (+ Making Ten)	*First Addend Unknown Problem:* __ + 12 = 19 12 = 10 + 2; 10 + 9 = 19; 9 – 2 = 7 *Sum Unknown Problem:* 12 + 9 = __ 12+9 = (10 + 2) + 9 = (9 + 2) + 10 = 11 + 10 = 21	Fuat Ayaz Irmak Tilbe
Dealing with Larger Ones (+ Making Ten + Ignoring Tens)	Sum Unknown Problem: 12 + 9 = __ 12 + 9 = (10 + 2) + 9 = (10 + 9) + 2 = 19 + 2 = (19 + 1) + 1 = 20 + 1 = 21 First Addend Unknown Problem: __ + 12 = 19 12 = 10 + 2; (10 + 7) + 2 = 19 [7 + 3 = 10 → 7 + 2 = 9 because 10 – 1 = 9]	Fuat Ayaz Zehra Metin
Ignoring Tens and Using Number Bonds (Number Combination Families)	*First Addend Unknown Problem:* __ + 12 = 19 19 → 12; 9 – 2 = 7; 19 – 12 = 7	Metin
Doubling and Add One	*Second Addend Unknown Problem:* 6 + __ = 13 (6 + 6) + 1 = 12 + 1 = 13; 6 +1 = 7	Zehra Metin

As shown in the table, we observed that students did not always use those strategies independently. Instead, *making (multiples of) ten* was often accompanied by *dealing with larger ones* and *using ten to compose a number*. Students' solutions involving a combination of several strategies in a compatible manner indicate the complexity and richness of students' ways of thinking in three types of addition problems. Here, it is also important to note that none of these strategies were taught to students during the instruction.

To interrogate further whether a particular type of problem favors certain strategies, we explored the code matrix presenting the landscape of students' strategies across problem types (see Table 3).

Table 3. Matrix of students' strategies across problem types

Strategies	Type of Addition Problems		
	First Addend Unknown [__ + 12 = 19]	Second Addend Unknown [6 + __ = 13]	Sum Unknown [12 + 9 = __]
Counting On	*	*	
Making (multiples of) Ten	*	*	*
Using Ten to Compose a Number	*		*
Dealing with Larger Ones	*		*
Ignoring Tens and Using Number Bonds	*		
Doubling and Add One		*	

Table 3 mainly presented one of the cross-comparisons held in the third phase of analysis. Hence, the matrix shows six strategies that the selected first-grade students used to solve three addition problems of different kinds and indicates a range of strategies varying across types of addition problems. For instance, *doubling and add one* strategy was observed only in the second addend unknown problem in which the first addend was 6 and the sum was 13, and therefore students could see that 13 was the double-plus-one of 6. Similarly, in the sum unknown problem, students' strategy choices were *making (multiples of) ten, using ten to compose a number,* and *dealing with larger ones.* One of the addends [9] in the sum unknown problem was close to *ten, which* might direct the students to use these strategies. On the other hand, all strategies, except *doubling and adding on*, were observed in the first addend unknown problem. Hence, this variation across the type of addition problems highlighted the role of the structure of the problems and the numbers involved in the problems, and therefore, we turned our attention to those and other factors influencing the first-grade students' strategy choices in addition word problems.

Factors Influencing Students' Strategy Choices in Addition Problems

We want to note that the findings presented in this section were drawn from a series of cross-comparative analysis, which was mainly driven by Baroody and Ginsburg's framework (1986) on the factors influencing students' strategy choices but open to additional factors that emerged.

Semantic Structure of the Problem

According to Baroody and Ginsburg (1986), the semantic structure of the problem is related to the nature of the situation described in the problem, such as a binary conception of a combine situation or a unary conception of a change situation. In addition to this interpretation of the semantic structure of the problems, we considered addition word problem types denoting another semantic structure in relation to the addition operation: first addend unknown structure, second addend unknown structure, and sum unknown structure. Hence, each problem type signifies a particular structure of the addition operation.

The first addend unknown problem that our participating students worked on the involved unary conception of a change situation because the number of students in the classroom became 19 when 12 more came in. Tilbe, who used the *counting-on strategy* to solve this problem, counted on the second addend until she reached the sum. The following dialog shows Tilbe's reasoning.

Interviewer: *How did you find the number of students in the class at the beginning?*

Tilbe: *I keep 12 in my mind. 12-13-14-15-16-17-18-19 (by showing her fingers)*

As can be seen from this brief exchange, Tilbe was comfortable with starting the counting by ones from the given addend [12] – no matter if it is the first or second addend – until she reached the sum [19]. Figure 2 shows her addition sentence in this problem.

Figure 2. Tilbe's addition sentence on the first addend unknown problem

Even though 12 was the second addend given in the problem, Tilbe counted on 12 and wrote 12 as the first addend in her addition sentence, compatible with her counting-on operation. Her addition sentence indicated that Tilbe conceptualized the unary change in one set (i.e., the number of students in a class) without considering the order of the addends. In this regard, the situational aspect of the semantic structure (i.e., the unary conception of a change situation) implicitly helped her to informally make sense of the commutative reasoning in addition operation.

The second addend unknown problem also involved the unary conception of a change structure because the number of Sude's toys was changed after her mom gave her some more. Fuat, who used *making (multiples of) ten* strategy to solve this problem where the first addend [6] and the sum [13] were given, and the second addend was asked. He expressed his solution as follows:

(Fuat quickly responded as 7 to the question)

Interviewer: *How did you find seven so quickly?*

Fuat: *I found it like that. I had 6. I added 4 [to 6] to make 10. Then, I added 3 [to 10] to reach the sum [to 13]. Then I counted 7.*

As seen in his explanation, he added 4 to the given addend [6] to get ten, using the number pairs of 10 as 6 and 4. He then used ten as a benchmark again to construct 13, for which he needed to add 3. In the final stage of his solution, he added 4 and 3, the numbers he added stepwise to reach 13. In this solution with *making (multiples of) ten* strategy, students engaged in composing the given addends in a compatible manner to reach the given sum. Although this strategy seems basic, it is, in fact, an elegant strategy that requires relational thinking among the numbers. Fuat wrote an addition sentence shown in Figure 3.

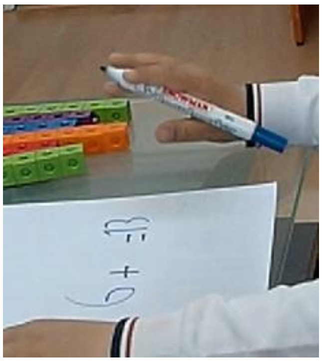

Figure 3. Fuat's addition sentence on the second addend unknown problem.

Fuat's strategy was influenced by the situational aspect of the semantic structure due to the unary change situation of the problem. Fuat's addition sentence also explicitly showed another aspect of the semantic structure of the second addend unknown problem that emerged from our students' work. Specifically, his addition sentence represented the known addend as the first number and the unknown addend as the second number. He wrote this addition sentence considering the order of the numbers given in the problem, another aspect of the semantic structure. To summarize, his strategy was shaped by the order of the numbers given in the problem because he started with the first addend [6] and aimed to construct the sum [13]. To identify the unknown addend, he used 10 as a benchmark, recalling the pairs of numbers making 10.

Cognitive Economy

Baroody and Ginsburg (1986) argued that students tend to prefer cognitively fewer demanding strategies or mental operations while solving problems. One of the main factors influencing students' choices of *dealing with larger ones* and *ignoring tens and using number bonds* was the cognitive economy. Four of the six participating students (Fuat, Ayaz, Zehra, and Metin) used this strategy that served for the cognitive economy because first adding larger ones to ten and then adding smaller ones to a teen number was easier for him. For example, in the sum unknown problem (12 + 9 = __), the exchange between Ayaz and the interviewer shows how he dealt with the larger ones before the smaller ones.

Interviewer: *How did you find the answer?*

Ayaz: *I took out 2 from 12 and then added 9 [to 10]. I added 1 more and 1 more. [that makes] 21.*

Interviewer: *I see you split 12 as 10 and 2 and added 1. That makes 20, and then?*

Ayaz: *I added 1 more. I mean I moved 9 to here (showing 10) and added 2 from 12 one by one.*

Interviewer: *So, what is your answer?*

Ayaz: *21 trees*

As seen in Ayaz's explanation, he first decomposed the first addend [12] into 10 and 2, and then added the second addend [9] that has larger ones compared to the ones in the first addend [2]. As the last step of the solution, he added the remaining ones from the first addend to the sum. Hence, Ayaz used *making (multiples of) ten* and *using ten to compose a number* strategies adaptively to invent this strategy. Besides, Ayaz's strategy implicitly involved a series of associative and commutative reasoning (i.e., (10+2) + 9 = 10 + (2+9) = 10 + (9+2) = (10+9) + 2). Hence, to deal with larger ones, he needed to decompose one of the numbers into tens and ones and consider the numbers associatively and commutatively, even though he did not formally know the principles of associativity and commutativity.

In the first addend unknown problem (__ + 12 = 19), Metin ignored tens on the second addend [12] and the sum [19]. This strategy allowed him to transform the problem into a single-digit first addend unknown problem (i.e., __ + 12 = 19 ←→ __ + 2 = 9). In other words, having the same number of tens in both the given addend and the sum allowed Metin to reduce the cognitive load of the problem by *ignoring tens*. Even though only one student demonstrated this strategy, it was worth understanding its components because it worked well for him to solve the first addend unknown problem. The following exchange between Metin and the interviewer shows how ignoring tens and focusing on the ones worked for him to solve the problem.

(Metin immediately stated "seven")

Interviewer: *How did you find 7 so quickly?*

Metin: *I took out 2 from 9. That's how I got 7.*

Interviewer: *Why did you subtract 2 from 9 then?*

Metin: *Because that was the easiest and quickest way.*

Interviewer: *So what happened to the 1s in 12 and 19?*

Metin: *There is no need to think about them.*

Interviewer: *Why?*

Metin: *Because we are going from 12 to 19.*

As seen in Metin's explanation, he realized that both the given addend and the sum have the same number of tens and, therefore, can be ignored to find the missing addend. This strategy may seem simple but requires a relational understanding between the addends and the sum. More specifically, Metin recognized that ignoring the same number of tens both in addend and in the sum would keep the balance, the equality, which is a foundational insight for early algebra.

In addition, we observed that *making ten* and *using ten to compose a number* strategies served as conceptual intermediate steps contributing to the cognitive economy. Ayaz and Fuat demonstrated *using ten to construct a number* strategy in more complex ways. In the first addend unknown problem (__ + 12 = 19), Ayaz made the following explanation that we found quite advanced.

Interviewer: *How did you find the answer?*

Ayaz: *I took out 2 from 12 and put 9 to that (10) [that would make 19].*

Interviewer: *Okay, you reached 19, but how did you get 7?*

Ayaz: *I took out 2 [that was left from 12] from 9 and got 7.*

In this strategy, Ayaz did not simply ignore the tens; rather, he mentally did the following operations: *(i)* use number combinations of 12 to decompose 12 into 10 and 2, *(ii)* to get 19 (i.e., the sum), add 9 (i.e., the number of ones in the sum) to 10 that was decomposed from the given addend (i.e., 10 of the 12), *(iii)* adjust the number of ones by subtracting the number of ones in the given addend from the number of ones in the sum; that is, 9-2=7, and *(iv)* conclude that the adjustment result [7] needs to be added to 12 to reach 19. As Ayazcan be seen, Ayaz's way of thinking is quite advanced.

On the other hand, Ayaz was not the only student who solved the same problem through advanced reasoning. The exchange between Fuat and the interviewer illustrates another sophisticated way of thinking.

Fuat: *If we add 7 to 12, that makes 19; therefore, there were 7 students at the beginning. I took out 2 from 12 and added 7 to 10. That made 17, and then I added 2, which made 19.*

Interviewer: *Sounds good. Can you tell me how did you find 7?*

Fuat: *I thought that 7 and 2 would make 9, and so I understood that it would be 7.*

Interviewer: *How do you know that 7 and 2 make 9?*

Fuat: *It is because 7 and 3 make 10, and 9 is one less than 10 because 9 comes before 10 [so 7 and 2 makes 9]. So, I added 2 and 7 and found 9. There is also 10 here (in twelve), and I added 10 and got 19.*

Similar to Ayaz, Fuat started by decomposing the second addend [12] into 10 and 2. He quickly found that the missing addend would be 7 because he immediately recognized 9 as 7 and 2, and that immediate recognition arose from his number combination knowledge of 10 (i.e., since 7 and 3 makes 10, 7 and 2 makes 9). So, he could reason that 7+2 needs to be added to 10 to get 19, and therefore 7 needs to be added to 12 to get 19. His recognition of 9 as 7 and 2 allowed him to bypass the second step that Ayaz followed in his strategy, which was possible with Fuat's fluent use of number combination knowledge. In other words, quick recognition of the relationship between numbers served to deflate cognitive load in selected strategies.

In the sum unknown problem (12 + 9 = __), Metin's solution involving *making (multiples of) ten* strategy is given below.

Interviewer: *How did you find the answer?*

Metin: *I subtracted 1 from 12. I got 11 then. I added 9 [to 11] and got 20. Then, 1 added 1 more [to 20] and obtained 21.*

Hence, he first split the first addend [12] into 11 and 1; because it would be easier to combine 11 and the second addend [9] to get multiples of ten. That is, Metin quickly transformed the first addend into a compatible number. By keeping 1 in mind, he combined 11 with the second addend and got 20. His knowledge based on the number combinations forming multiple of ten helped him to figure out the sum. Lastly, he added 1 to 20 to find the result.

As a second example of the solution to the sum unknown problem, we provide Irmak's case.

Interviewer: *Could you explain your solution to me?*

Irmak: *I think 12 as 10 and 2. I added 2 to 9 and got 11 then.*

Interviewer: *As far as I see, you wrote 12 there (showing the paper).*

(She wrote 11 near 12. Then, struck out 12)

Interviewer: *Yes, you added 2 to 9 and got 11, then?*

Irmak: *Then, I took this ten (showing 12) and join to this sum [to 11]. I mean, I added 10 and sum [11].*

As can be seen from the given dialog, Irmak first decomposed 12 into 10 and 2 by taking into account the place value concept and ten as a benchmark. Then, she composed ones and ended up with a total of 11, which was seen in her written work below (Figure 4).

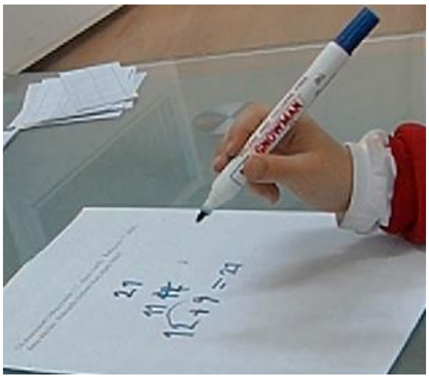

Figure 4. Irmak's addition sentence on the sum unknown problem

As a last step, she added 10, that she obtained from the first addend, to that total. When Irmak was probed about how to add 10 and 11, she split the number into tens and ones and first added tens together and then ones. We could deduce from her reasoning that she could work with ones and tens regardless of the order. That is, she used ten first to compose 11 (i.e., (9+1) + 1) and then to compose 21 (i.e., (10+1) + 10 = (10+10) + 1). Hence, Irmak's strategy involved *using ten* to compose 21. In this problem, Irmak demonstrated that she could construct any number using ten in four ways: (a) *decomposing a number into tens and ones* (i.e., 12 is 10 and 2), (b) *making ten* considering the number pairs of ten (i.e., 9 and 1 makes 10), (c) *making multiples of ten* (i.e., 10 and 10 makes 20), and (d) *using multiples of ten to construct any number* (i.e., 21 is 20 and 1). These ways of flexibly using ten to construct any number allowed her to add 11+10 by mentally considering that (10+1) + 10 was the same as (10+10) +1, which implies informal associative and commutative reasoning. Although we do not claim that she developed those properties of addition, we observed that her *using ten* strategy flexibly allowed her to think associatively without being formally aware of it. Furthermore, her construction of a number using ten in multiple ways contributed to the cognitive economy.

Not Number Size but Numbers Included in the Problem: An Adapted Factor for Strategy Choices

Although Baroody and Ginsburg's strategy choice framework (1986) included problem size as the third influential factor for students' strategies, we observed in our study another distinct but related factor that we called "numbers included in the problem". In other words, not necessarily the size of the numbers, even the numbers smaller than 20 but chosen intentionally for the problem had the potential to influence students' strategy choices.

We showed above that *making ten* strategy was the most basic and conceptual strategy, serving as a ground for other strategies, such as *using ten to compose a number* and *dealing with larger ones*. We observed that the choices of the numbers in problems influenced students' choice of the *making ten* strategy. For instance, in the second addend unknown problem (6 + __ = 13), the first addend [6] was given closer to 10 intentionally, and Fuat could easily recall his number combinations knowledge "6 and 4 makes 10." Afterward, he used ten to construct a teen number [13], which was another number combinations practice he had while learning teen numbers (i.e., 10 and 3 make 13). Hence, intentionally including particular numbers allowed five students in this study to think relationally and transitively (i.e., 10 is 4 more than 6, 13 is 3 more than 10, and so 13 is 4+3 more than 6) and played a role in their strategy choices. In the same problem, Metin and Zehra used doubling and adding one, relying on their number combination knowledge. Students recalled that "6 and 6 make 12" and increased the addend by one (i.e., 6+1), which resulted in an increase in the sum by one (i.e., 12+1). Hence, the numbers included in the problem played an important role in students' choice of *doubling and add one* strategy and also oriented them to think relationally between the size of the addend and the size of the sum.

In the first addend unknown problem (__ + 12 = 19), we demonstrated Ayaz's and Fuat's advanced solutions above. Those solutions were mainly based on *making ten* and *using ten to compose a number*, which was highly influenced by the numbers included in the problem. More specifically, having both the known addend [12] and the sum [19] bigger than ten allowed students a series of composition and decomposition and handling the number of ones from the addend and the sum differently. Thus, the choices of the numbers in the problem not only influenced students' strategy choices but also broadened the range of strategies and so increased the level of complexity in those strategies. In the same problem, we also observed Metin, who chose to ignore the tens and transformed the problem into a single-digit addition problem. Since both the given addend [12] and the sum [19] involved the same number of tens, Metin ignored these tens to reduce the cognitive load of the problem, which was possible by intentional choices of the numbers involved in the problem. Similarly, the numbers given in the sum unknown problem (i.e., 12 + 9 = __) influenced all six students' strategy choices. In other words, intentionally having one number slightly more than ten [12] and another number slightly less than ten [9] oriented students to the *using ten* strategy.

Hence, the numbers chosen for the problem played a role in students' strategy choices even though those numbers were not large numbers or one of the addends was not considerably larger than the other one (i.e., problem size). Since Baroody and Ginsburg's strategy choice framework (1986) mainly focused on the problem size, our analysis of *the numbers included in the problem* did not completely match with the researchers' third factor. Therefore, we extended this factor to the numbers involved in the problem but not necessarily to the size of the numbers.

Discussion and Conclusion

Above, we portrayed the factors that played role in the selection of first grade students' strategies in addition word problems. The strategies our students developed in this study (see Table 3) were compatible with the ones observed in other studies (e.g., Baroody et al., 2015; Clements et al., 2020; Fuson, 1992; Guerrero & Palomaa, 2012; Paliwal & Baroody, 2020), and, in fact, went beyond those in the sense that our students combined some of the strategies flexibly, which appeared an important aspect of students' strategy choices (Russo & Hopkins, 2018; Sunde & Sunde, 2019).

Our findings supported Mulligan's argument (2004) that some strategies seemed to be more effective in particular problem types. As we presented in Table 3, the first addend unknown problem elicited more strategies than other problem types. Considering that the first addend unknown problem was the one that students found challenging more than other types of addition problems (Van De Walle, Karp, & Bay-Williams, 2013), we argue that students' struggle could produce more strategies because they needed to reduce the cognitive load of the challenging problem. In this respect, students' struggle drove the cognitive economy. Our findings, on the one hand, were not parallel to and, on the other hand, supported the findings presented in Kayhan-Altay's study (2023). Specifically, the researcher pointed out that 2nd, 3rd, and 4th grade students performed better in the sum unknown type of addition problems; however, in our study, the first-grade students developed more strategies in the first addend unknown problems compared to the sum unknown structure, which presented a contradiction. On the other hand, this indicated students' struggle in the first addend problem compared to the sum unknown problem. This supported Kayhan-Altay's (2023) findings that students performed better in the sum unknown problem structure.

We further elaborated on the cognitive economy perspective in relation to conceptual and relational understanding because they contributed to lowering the cognitive demand of the problem. The flexible use of number combinations, often number pairs of 10, cognitively supported students' mental strategies, which turns the complex solution into a relatively less demanding process for some students. Therefore, those students tended to operate composing and decomposing on number combinations in their strategies as a less cognitively demanding procedure. Hence, we claim that number combination knowledge created a conceptual link for students to think relationally, which indeed reduced the workload for students. Considering that number combination knowledge is based on part-part-whole relations of numbers, our observation is aligned with Ding and Auxter's argument (2017, p. 88): "A more complete understanding of the part-whole structure may provide flexibility when dealing with what is given and what is unknown." This finding also supported the studies claiming that relational understanding of numbers was a conceptual foundation for strategy development (e.g., Chu et al., 2018; Kayhan-Altay, 2023; Schiffman & Laski, 2018).

It is interesting and promising result that the first-grade students in this case investigation did not use traditional algorithms to solve the addition problems. Students' reluctancy of using standard algorithm might stem from nature of HLT that we used during the main study. Our hypothetical learning trajectory mainly focused on enhancing students' conceptual understanding through the use of alternative strategies in solving addition and subtraction problems. As also mentioned in the instruction description, students in this study were formally introduced to horizontal and vertical addition notations; however, they all preferred to use horizontal notation and did not use standard algorithm. This result contradicts Kayhan-Altay's study in which 2nd, 3rd, and 4th graders tended to use mostly standard algorithm. However, this might be because of the numbers involved in the problems. Specifically, the numbers involved in the problems were less than 20, as suggested by the first-grade mathematics curriculum (MoNE, 2018), and therefore, students might not need to use the standard algorithm. Another reason might be that as grade levels increase, students' experiences with the standard algorithm increases; and therefore, students get more familiar with the method. Güç and Hacısalihoğlu (2016) claimed that students are keen to use familiar strategies therefore the more familiar they get with the traditional algorithm in the upper grades, the more they might use it. Bütüner (2020) found that Turkish 3rd and 4th-grade mathematics textbooks included fewer opportunities for strategy development and more included standard algorithm methods compared to Singaporean textbooks. This exposure in upper primary years might also influence students' tendency toward standard algorithms, as the first graders in this study did not yet develop this familiarity with standard algorithms, and therefore, they might lean toward using various strategies.

We also want to note that the strategies used by first grade students in addition problems possessing different semantic structures highly based on composition and decomposition; that is, the part-part-whole relation existed in the number bonds. Although it might be seen as trivial, Güç and Hacısalihoğlu (2016) reported that even middle school students had difficulty in using decomposition in mental addition problems. On the other hand, once students conceptualized the part-part-whole relation, it contributes to the cognitive economy to a greater extent, and students often develop strategies involving composition and decomposition, as we observed in our study with the case of selected first-grade students and also in Duran et al.'s (2016) with middle school students.

Another major contribution of this study is the extension of Baroody and Ginsburg's conception of the semantic structure of the problem. We observed that the semantic structure was not only related to the structure of the situation but also to the structure of the addition operation – whether it is a first-addend, second-added, or sum unknown structure – and the order of the numbers given in the problem. Therefore, we extended Baroody and Ginsburg's conception by layering out the aspects of the semantic structure as (i) the situational aspect of the semantic structure, which was the factor in the original framework, (ii) the operational aspect of the semantic structure, and (iii) the number-order aspect of the semantic structure. While the situational aspect of the semantic structure is about whether the situation is a binary/unary combine or change situation, the operational aspect of the semantic structure is related to the known and unknown quantities in the addition sentence, and finally, the number-order aspect indicates whether the numbers indicating the first addend and the second addend are stated respectively in this order in the word problem. Particularly, we observed the number-order aspect in students' addition sentences. In writing addition sentences, the order of the numbers stated in the problem influenced students' strategy choices.

Baroody and Ginsburg's (1986) framework involved a third factor called the problem size. Other studies also confirmed the role of the size of the numbers in enhancing students' strategy choices, particularly using larger numbers compared to the single-digit number (e.g., Guerrero & Palomaa, 2012; Verschaffel et al., 2007). For instance, having one of the addends larger may lead students to choose counting on-from-the larger strategy instead of counting on-from-the first strategy for 4+22 (Baroody & Ginsburg, 1986), in which the ultimate goal is reducing the cognitive load. Therefore, we considered the problem size factor related to the cognitive economy factor. Since our addition problems Involved numbers between 1-20 due to curriculum restriction (MoNE, 2018) and the numbers given in the problems were close to each other, we could not identify problem size as one of the influential factors for our students' strategy choices. Being bounded by the national curriculum and limiting the numbers between 1-20 could be why problem size did not appear as influential in our first-grade students' strategy choices. However, our analysis indicated a related factor that emerged from our students' work; that is, the numbers involved in the problem. Therefore, we presented an adapted factor that is related to the numbers involved in the problems but not necessarily considering the size of the numbers. In all three types of addition problems, numbers given in the problem appeared to be an influential factor for students' strategy choices for almost all strategies (except counting-on). In other words, we observed that the numbers that were intentionally used in the problem had the potential to lead to students' strategy choices. Although Pongsakdi et al.'s study (2020) showed that numerical factors were not deterministic for creating a challenging problem, arithmetic skills appeared to be an important factor in solving word problems. Considering Baroody and Ginsburg's (1986) third factor, the result of Pongsakdi et al.'s study (2020), and the result of our study altogether, we may articulate that numbers involved in the problem might not necessarily produce easy or difficult problems but might lead students' arithmetic strategies aiming to lower the cognitive load of the problem.

Lastly, we found it noteworthy to mention the learning environment that supports students' development of a variety of strategies without explicit instruction on a particular strategy (Schiffman & Laski, 2018; Sievert et al., 2019; Sunde & Sunde, 2019; Torbeyns et al., 2005). Similarly, research indicated that resources such as textbooks and materials played a role in strategy development (Kalaycıoğlu Akis & Şahin, 2023; Üstündağ & Özçakır Sümen, 2023). In this regard, this study involved an intervention that focused on number combination families using the number bonds structure as cognitive materials, which supported first-grade students' flexibility in strategy choices. During the intervention, we did not prompt students to use certain strategies or give them explicit strategy-based instruction, albeit focusing on the number combinations opened the doors for the first-grade students' use of multiple strategies or combinations of strategies, which supports Mulligan's argument in this regard (2004). Thus, being involved in treatment might have influenced students' thinking and enhanced their learning of arithmetic (Chu et al., 2018; Clements et al., 2020), which in turn made them competent and flexible in using these strategies.

Implications and Recommendation

This study indicated several implications for mathematics education practices and related research. First, students' use of strategies in addition problems was mainly based on their knowledge of number pairs. Therefore, we suggest teachers ensure that students possess a stable and flexible number combinations knowledge for enriching students' strategies in addition word problems. Second, the participating students' strategy choices were influenced by the semantic structure of the problem, cognitive economy, and the numbers included in the problem. Among these factors, the cognitive economy appeared in our study as connected to students' use of number combinations as a conceptual link for strategy development, which consolidates the above-mentioned implication about supporting students' number combinations knowledge.

In addition, the numbers involved in the problems highly influenced students' strategy choices. In this respect, we suggest teachers be aware that not only problem types but also the numbers involved in the problems that played role in students' strategies in addition problems. This informs curriculum developers and textbook writers about considering the strategy choice framework in writing mathematics problems. We also recommend teachers be aware of possible factors influencing students' strategies and how they may regulate those factors for enhancing students' strategic competence since teachers are the key for developing students' strategic competence (Durkin et al. (2017).

On the other hand, our study was limited to only six first-grade students and three addition word problems. We recommend further investigating the factors influencing students' strategy choices with a larger group of students and with a variety of problems in each type of addition problem. Lastly, we suggest investigating students' strategies and the factors influencing the strategy choices in subtraction problems and comparing the underlying factors with those that emerged in this study.

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