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Original Article

Middle School Students' Mathematical Resilience and Perceptions of Mathematics: A Cluster Analysis Approach *

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Abstract

The primary purpose of the present study was to examine whether Turkish middle school students' perceptions of mathematics teachers, perceptions of mathematics learning, and their mathematical resilience can be used to divide students into clusters with similar profiles. The sample consisted of middle school students with ages ranging from 11 to 15. Two-step cluster analysis was used to create profile groups. As a result of cluster analysis, two distinct profiles were revealed. The results showed that clusters -formed in the present study differentiated according to students' perceptions of mathematics teachers, learning mathematics, their mathematical resilience, and their mathematics achievement. Students in cluster 1 have higher mathematics achievement, value mathematics, perceive mathematics teachers as knowledgeable and supportive people, perceive learning mathematics as a fun process requiring effort. Students in cluster 2 have lower mathematics achievement. They perceive mathematics teachers as the source of anxiety and learning mathematics as a difficult process. They also have low belief levels that mathematics ability can be improved in comparison to cluster 1.

Keywords

Students' perceptions of learning mathematics

Students' perceptions of mathematics teachers

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Introduction

Mathematics achievement is an important issue that is addressed in numerous educational studies. Previous research emphasized that mathematics achievement is significantly associated with students' perceptions of mathematics and mathematics teachers and draws attention to the importance of the perceptions in mathematics achievement (Daher, Anabousy, & Jabarin, 2018; Yalçın, 2012). Of these studies, a large body of research is variable-centered studies (e.g., Arıkan, van de Vijver, & Yağmur, 2016; Daher et al., 2018; Helm, Arens, & Möller, 2020; Sakiz, Pape, & Woolfolk-Hoy, 2012). Even though the variable-centered analytic models such as correlation, regression are well suited for questions that concern the relative importance of predictor variables in explaining variance in outcome variables (Laursen & Hoff, 2006; Muthen & Muthen, 2000), they do not provide information about student profiles (Litalien, Gillet, Gagne, Ratelle, & Morin, 2019). In contrast, person-centered analyses

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such as cluster analysis focus on relationships among individuals from a holistic perspective, rather than following a traditional variable-oriented approach that investigates linear relationships between variables (Bergman, 1998). The goal is to group individuals into categories (i.e., profiles), each of which contains individuals who are similar to each other and different from individuals in the other categories (Muthen & Muthen, 2000). Thus, the combination of variables in the cluster of participants with similar characteristics can be displayed.

In previous person-centered studies in mathematics education, researchers focused on clustering students by various variables such as their achievement emotions, achievement emotion regulation strategies, competence in mathematics learning (e.g., Abd-El-Fattah, 2018; Hanin & Van Nieuwenhoven, 2019), self-concepts of ability (e.g., Umarji, McPartlan, & Eccles, 2018), mathematics anxiety (e.g., Mammarella, Donolato, Caviola, & Giofrè, 2018), and mathematics motivations (Lazarides, Dietrich, & Taskinen, 2019). However, to the authors’ knowledge, there is no research examining the students' perceptions of mathematics teachers, perceptions of learning mathematics, and their mathematical resilience by using a person-centered approach. Furthermore, although it is well known that students' perceptions of mathematics teachers and mathematics courses are related to mathematics achievement (Sakiz et al., 2012), how these variables combine has been yet unknown. Using a person-centered analysis approach, it could be revealed how students who have similar characteristics according to their perceptions of mathematics teachers, perceptions of mathematics learning, and their mathematical resilience get together. The findings obtained from the present study could be useful for many mathematics teachers to identify patterns related to their students' perceptions of mathematics teachers, perceptions of mathematics learning, and their mathematical resilience. So, it could be possible to define instructional strategies that can change their students' perceptions to the positive side and improve their mathematical resilience if patterns are available. Therefore, the current study examines the middle school students' perceptions of mathematics teachers, their perceptions of learning mathematics, and their mathematical resilience using a person-centered approach. Accordingly, two research questions have been formulated:

1. Can students' perceptions related to mathematics teachers, perceptions of mathematics learning, and their mathematical resilience be used to categorize students into clusters with similar profiles?
2. If profiles can be established, are these profiles associated with students' mathematics achievement?

Conceptual Framework

Perceptions about Mathematics Teachers

Previous studies emphasized that students' perceptions of mathematics teachers have an important role in mathematics achievement (Mutadi & Ngirande, 2014; Yalçın, 2012). Previous studies also showed that students' perceptions of mathematics teachers also have a role in the variables closely related to their mathematics achievements, such as effort, self-efficacy, academic emotions, attitudes, motivations, and perceptions that mathematics is useful and valuable (Bawuah, Sare, & Kumah, 2014; den Brok, Fisher, & Scott, 2005; den Brok, van Tartwijk, Wubbels, & Veldman, 2010; Federici & Skaalvick, 2014a, 2014b; Jang, Reeve, & Deci, 2010; Putwain & Symes, 2011; Reddy, Rhodes, & Mulhall, 2003; Sakiz et al., 2012). For example, previous research results in perceived teacher closeness revealed that students who perceived closeness in the relationship with their teacher more approach their mastery goals (e.g., Thijs & Fleischmann, 2015). Research on teacher unfairness showed that perceived teacher unfairness is negatively associated with students' self-concept, interest in mathematics, and perceptions of the importance of doing well on mathematics tasks (e.g., Helm et al., 2020). In addition, many studies highlighted the protective role of perceived teachers’ emotional support on school burnout (e.g., Romano, Angelini, Consiglio, & Fiorilli, 2021). Although these research results showed the importance of students' perceptions of mathematics teachers on many variables, understanding students' perceptions of mathematics teachers is not straightforward. Because some studies revealed that students' perceptions of teachers could differ according to students' characteristics. For example, in a study conducted by Zee and Roorda (2018), whereas shy children had less close and more conflictual student-teacher relationships, anxious children had more conflictual and dependent student-teacher relationships. These findings also suggest that students' perceptions of teachers could alter in terms of their different characteristics, and therefore students' perceptions of teachers should be addressed considering students' different characteristics.

The current study covers three main student perceptions of mathematics teachers, which were determined in comprehensive research by Yalçın (2012). According to the results of Yalçın's study, some students perceive mathematics teachers as supportive people, while some perceive them as a source of anxiety. Whereas some students perceive mathematics teachers as knowledgeable people who make calculations fast like a machine, who have a strong memory and extensive knowledge in mathematics. Furthermore, Yalçın's study results also revealed that these perceptions significantly related to student mathematics achievement and attitudes toward mathematics (Yalçın, 2012). Therefore, in the current study, students' perceptions of mathematics teachers were taken as possible factors that could differentiate students' profiles in mathematics courses.

Perceptions about Learning Mathematics

In the current study, students' perceptions of learning mathematics are considered the other possible factors related to students' profiles. Yalçın's (2012) framework of students' perceptions of mathematic learning was adopted to assess students' perceptions of learning mathematics. According to this framework, there are three main student perceptions of mathematics learning (Yalçın, 2012): the perception that learning mathematics is a challenging process, the perception that learning mathematics is a fun process, and the perception that effort is necessary for learning mathematics.

The students who have a perception that learning mathematics is a challenging process perceive mathematics learning as a challenging, complicated process that is full of obstacles. This perception reflects the negative aspect of perceptions related to mathematics courses. According to Yalçın's research results, students' perception level that mathematics learning as a challenging process is negatively related to their mathematics achievement (Yalçın, 2012). In also other studies on students' perception of difficulty related to mathematics, results showed that the students' perceptions related to mathematics are negatively associated with the effort in mathematics and perceptions about being successful in mathematics (Hannula & Laakso, 2011; Ma, 1997; Mutadi & Ngirande, 2014; Yalçın, 2012).

Another important perception related to learning mathematics in Yalçın's study is that mathematics learning is fun. Yalçın's research results showed that there is a positive relationship between students' perceptions that mathematics is a fun process and their mathematics achievement (Yalçın, 2012). Perceiving mathematics learning as a fun process contains enjoyment emotion. In the context of the control value theory of achievement emotions, it is known that emotions and learning are related to each other (Pekrun, 2000, 2006). Indeed, numerous studies have shown a positive relationship between learning-related positive emotions and achievement (Goetz, Frenzel, Hall, & Pekrun, 2008; Ma, 1997). Furthermore, research results revealed that learning-related emotions such as enjoyment is not only the significant predictor of achievement but also positively associated with several variables related to mathematics achievements such as self-regulation, strategy use, motivation, achievement goals, and activation of cognitive resources (Pekrun, Elliot, & Maier, 2006; Pekrun, Goetz, Titz, & Perry, 2002).

The last perception in the perception framework related to mathematics learning in Yalçın's study is the perception that mathematics learning is a process that requires effort (Yalçın, 2012). Given that the effort was defined as "the amount of time and energy that students expend in meeting the formal academic requirements" (Carbonaro, 2005), it can be expressed that effort is crucial and necessary in mathematics achievement. Indeed, research results revealed that effort is the significant predictor of achievement and is positively associated with various variables closely related to mathematics achievements such as mastery goal orientations, utility value, and competence beliefs (Carbonaro, 2005; Chouinard, Karsenti, & Roy, 2007). The research results mentioned above have shown that three main perceptions about mathematics learning (i.e., challenging process, fun process, and a process requiring an effort) were the crucial variables that can differentiate to form students' perception profiles. Hence, in the current study, different students' perceptions of mathematics learning were considered possible factors that could determine students' profiles in mathematics courses.

Mathematical Resilience

Mathematical resilience has been defined as "a learner's stance towards mathematics that enables student to continue learning despite finding setbacks and challenges in their mathematical learning journey" (Johnston-Wilder & Lee, 2010). The resilient students in mathematics were conceptualized as students who find mathematics valuable, think that mathematics is a process that requires effort and mathematics can be developed, keen to learn mathematics despite its difficulties, has the confidence to learn mathematics (Hutauruk & Priatna, 2017; Kooken, Welsh, Mccoach, Johnson-Wilder, & Lee, 2013; Kooken, Welsh, Mccoach, Johnston-Wilder, & Lee, 2016).

The present study adopted the mathematical resilience framework that Kooken and colleagues developed. As a result of several studies, Kooken and colleagues described mathematical resilience with three significant components: value, struggle, and growth. The value component refers to the extent to which students find studying mathematics important in attaining their current or future goals, whereas the struggle component refers to a student's perception that they sometimes must exert a great deal of effort because math can be a challenge to learn. Finally, growth is defined as the perception that people cannot improve their knowledge of mathematics. Students who have this perception believe that some people have a mathematics gene, and everyone does not learn mathematics (Kooken et al., 2016). In Turkey, Çağlayan (2018) and Gürefe and Akçakın (2018) used mathematical resilience scale developed by Kooken et al. (2016) in their study. Their research results revealed that the hypothesized three-factor structure was confirmed in the Turkish sample.

The relationships between mathematical resilience and various variables were examined in many studies. The research results demonstrated that mathematical resilience is positively correlated with mathematics achievement (Borman & Overman, 2004; Johnston-Wilder, Brindley, & Dent, 2014; Kooken et al., 2013, 2016) and is negatively correlated with mathematics anxiety (Cropp, 2017). Moreover, experimental research results showed that mathematics achievement could be improved with the development of resilience (Cropp, 2017; Johnston-Wilder & Lee 2010; Johnston-Wilder, Lee Brindley, & Garton, 2015). Based on these research results, it can be claimed that mathematical resilience could have essential implications for mathematics achievement. Hence, in the current study, mathematical resilience was considered an important factor that could affect students' profiles in mathematics course context besides students' perceptions of mathematics teachers and perceptions related to mathematics learning and included in the research.

Method

Research Design

The present research aimed to examine the middle school students' perceptions of mathematics teachers, learning mathematics, and their mathematical resilience using cluster analysis. Cluster analysis is employed to group individuals similar in a specific category and different from individuals in the other categories (Muthen & Muthen, 2000). Also, the current study examined whether clusters determined are differentiated according to math achievement using multivariate variance analysis. Therefore, the present study was designed using a descriptive research model.

Participants

The current study was carried out using a sample comprising middle school students attending the public schools in the North-West of the Black Sea region of Turkey. The sample was determined by using convenience sampling based on voluntary participation. Of the 850 participants, 497 (260 girls) participants fully completed the scales (61.5% response rate). The ages of the students ranged from 11 to 15 (M = 12.83, S = 0.93). The sample consisted of 197 sixth-grade, 158 seventh-grade, and 142 eighth-grade students (see Table 1).

Table 1. Sample information

	n	%
Gender		
Girl	260	52.3
Boy	237	47.7
Age		
11	20	4.0
12	193	38.8
13	147	29.6
14	125	25.2
15	12	2.4
Grade level		
6	197	39.6
7	158	31.8
8	142	28.6

Procedure

Before carrying out the study, approval was obtained from the Ethics Committee for Human Studies. Three scales used in the current study were administered to middle school students in the 2016-2017 spring semester by the first researcher in their course hour. The research aim was explained to the students before starting the scale implementation. The students were reassured that their answers would not be shared with others and affect their grades. The students were also informed that their course teacher would not reach any scale answers. The scale administration took 30 minutes.

Research Instruments

Metaphors Related to Mathematics Teacher Scale (MMTS)

The MMTS (Yalçın, 2012) is a self-report scale with 19 items used to measure students' perceptions of mathematics teachers through metaphors. The MMTS contains three first-order dimensions. The first dimension of the MMTS is a "supportive person" dimension, which has eleven items (e.g., "The mathematics teachers are like the guide because you can find your way through his/her guidance"). This dimension reflects students' perceptions of mathematics teachers as a guide who support them. In the present study, an item from the supportive person dimension is omitted from the scale because of a non-significant t value and low factor loading. The second dimension of the MMTS is the "knowledgeable person" dimension, which has five items (e.g., "The math teacher is similar to a calculator because he does a lot of calculations easily"). The knowledgeable person dimension measures the students' perceptions that mathematics teachers have a high level of knowledge and skill in mathematics. In other words, getting a high score from this dimension indicates that mathematics teachers are perceived as individuals with a high level of knowledge and skills in mathematics. The third dimension is the "source of anxiety" dimension and is measured with three items (e.g., "The math teacher is like thunder because it creates fear"). The source of anxiety dimension reflects the negative aspect of students' perceptions of mathematics teachers. The score in this dimension demonstrates a measure of the extent of what students perceive mathematics teachers as individuals who create a fearful environment and causes students to worry (Yalçın, 2012).

Responses on this scale are ranging from 1 to 5 (strongly disagree to strongly agree). In the current study, confirmatory factor analysis results with 18 items revealed that the first-order MMTS model fit the current data well ($\chi^2(132) = 408.99$, CFI = .96, TLI = .93, RMSEA = .06). Standardized parameter estimations ranged from .54 to .82. The alpha coefficients were computed as .92, .80, and .76 for a supportive person, knowledgeable person, and source of anxiety subscales, respectively (see Table 2).

Metaphors Related to Learning Mathematics Scale (MLMS)

The MLMS (Yalçın, 2012) is a self-report scale with 18 items used to measure students' perceptions of learning mathematics through metaphors. The MLMS consists of four factors: Perception related that learning mathematics is a challenging process is the first dimension of the MLMS, and it consists of five items (e.g., "Learning math is like going to space because it's hard to go"). This dimension measures the perception level of mathematics learning as a process that is arduous and full of obstacles (Yalçın, 2012). This dimension reflects the negative aspect of mathematics lessons. The second dimension of the MLMS measures perception that learning mathematics is a fun process, consisting of five items (e.g., "Learning math is like playing games because it is as enjoyable as playing games"). Besides, this dimension reflects the positive aspect of opinions about mathematics lessons; it also expresses the degree of enjoyment of mathematics lessons (Yalçın, 2012). The third dimension of the MLMS is perception related that learning mathematics is a process that requires effort, and it consists of five items (e.g., "Learning math is like realizing a dream because you need to strive to achieve"). This dimension reflects students' perception that effort is necessary to learn mathematics (Yalçın, 2012). In other words, students who have high scores in this dimension think that they should strive to learn math. Finally, the fourth dimension of the MLMS is perceptions related that learning mathematics is only a process, and it consists of three items (e.g., "Learning mathematics is like a seedling because it grows as it watered"). Because of the low-reliability result, this dimension was not considered in later analysis.

Responses on this scale are ranging from 1 to 5 (strongly disagree to strongly agree). Confirmatory factor analysis results revealed that the first-order MLMS model fit the current data well ($\chi^2(129) = 446.05$, CFI = .92, TLI = .91, RMSEA = .07). Standardized parameter estimations were ranged from .45 to .82. The alpha coefficients were computed as .88, .88, and .77 for challenging process, fun process, and process requiring effort subscales, respectively (see Table 2).

Mathematical Resilience Scale (MRS)

The MRS (Kooken et al., 2016) is a self-report scale with 24 items measuring mathematical resilience. The MRS consists of three factors: The first dimension is the value dimension, consisting of eight items (e.g., "Having a solid knowledge of mathematics helps me understand more complex topics in my field of study"). The value reflects a measure of students' perceptions that mathematics is important in attaining their current or future goals. The second dimension is struggling dimension, consisting of nine items (e.g., "Struggle is a normal part of working on math"). The struggle reflects perceptions that learning mathematics requires a great deal of effort. Finally, the third dimension is growth dimension, consisting of seven items (e.g., "If someone is not a math person, they will not be able to learn much mathematics"). The growth reflects the perception that people cannot improve their knowledge of mathematics. Therefore, gaining a low score in this subscale represents high mathematical resilience.

Responses on this scale are ranging from 1 to 5 (strongly disagree to strongly agree). Four items were removed from the scale because of their non-significant t values and low factor loadings. CFA results with 20 items show that first-order model of mathematical resilience had good fit to the current data ($\chi^2(167) = 453.22$, CFI = .90, TLI = .90, RMSEA = .06). Standardized parameter estimations were ranged from .42 to .77. The alpha coefficients were computed as .86, .74, and .70 for value, struggle, and growth subscales, respectively (see Table 2).

Table 2. Descriptive analysis and confirmatory factor analysis results

Scale	Subscale	M	S	χ^2(df)	CFI	TLI	RMSEA	SPE	α
MMTS				408.99(132)*	.96	.93	.06	.54-.82	
	Supportine person	44.41	8.05						.89
	Knowledgeable person	20.94	3.77						.80
	Source of fear	6.20	3.35						.76
MLMS				446.05(129)*	.92	.91	.07	.45-.82	
	Challenging process	15.02	5.93						.88
	Fun process	17.96	5.51						.88
	A process requiring effort	21.34	3.58						.77
MRS				453.22(167)*	.90	.90	.06	.42-.77	
	Value	33.42	5.85						.86
	Struggle	28.17	4.73						.74
	Growth	14.51	4.55						.70

Note. *p<.001; M = mean, S = standard deviation, MMTS = metaphors related to mathematics teacher scale, MLMS = metaphors related to mathematics learning scale, MRS = Mathematical Resilience Scale, χ^2 = chi-square, df = degrees of freedom, CFI = comparative fit index, TLI = Tucker Lewis index, RMSEA = root mean square error of approximation, SPE = standardized parameter estimation, α = alpha coefficient

Mathematics Achievement

Students' mathematics achievement was determined based on mathematics grades averages for the 2016-2017 academic year. Students' grades were taken from students' mathematics teachers in June 2017 by taking permission from the school principal. The grades ranged from 0 to 100. The mean of mathematics achievement was 84.64 (S = 14.91).

Data Analysis

Firstly, the missing data were removed from the data set. Then, factor structures of the scales were tested with confirmatory factor analysis by using AMOS 24 (Arbuckle, 2016). The maximum likelihood method was used for the confirmatory factor analysis parameter estimations. The alpha coefficients were calculated to get evidence of reliability for the subscales.

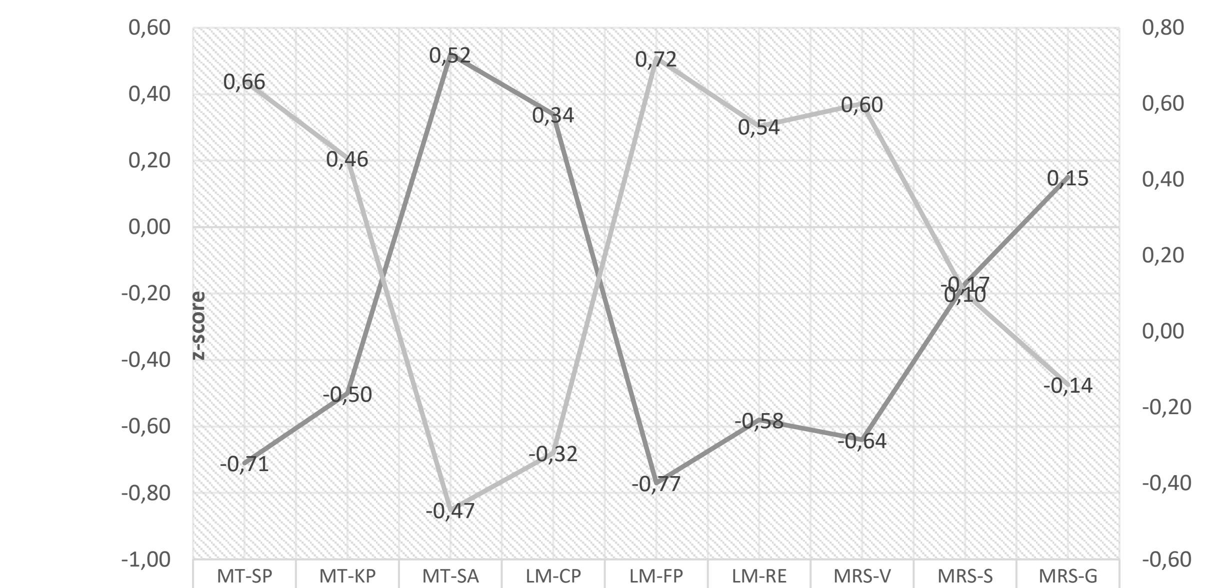
In the second step, the two-step cluster analysis was used to identify distinct profiles by gathering students with similar characteristics according to their perceptions of mathematics teachers, perceptions of learning mathematics, and their mathematical resilience level. Two-step clustering analysis consists of two-stage, as understood from its name. In the first stage, pre-clusters are created. At this stage, the Schwarz Bayesian Information Criteria (BIC) or the Akaike Information Criteria (AIC) are used to determine the cluster number. The purpose of the pre-clustering step is to reduce the size of the matrix containing the distances between all possible case pairs. Clusters are determined using pre-clusters and the agglomerative hierarchical clustering algorithm in the second step. The initial estimate is refined in the second step, finding the largest increase in distance between the two closest clusters at each hierarchical clustering step (Satish & Bharadhwaj, 2010). The primary reason for using the two-step cluster algorithm in the current study is that it automatically determines the number of clusters. Two distance measures were used in the two-stage clustering analysis: Log-likelihood and Euclidean distance measure. As the research variables are continuous in the present study, the clusters were determined by the Log-likelihood distance. Schwarz Bayesian Information Criteria were used to determine the number of clusters.

In the third step, a series of multivariate variance analyses (MANOVA) was conducted to examine the effects of age and gender on research variables. The results showed that the impacts of gender and age on the research variables were significant. Therefore, in the fourth step, multivariate analysis of covariance (MANCOVA) was used to test whether there was a significant difference between the clusters according to the research variables (cluster validity) and students' achievement, considering the impact of gender and age on research variables.

Results

Student Profiles

Cluster analysis was performed using middle school students' perceptions of mathematics teachers, perceptions of mathematics learning, and their mathematical resilience components to create student profiles. As a result of the two-step cluster analysis, students were separated into two clusters based on their perceptions of mathematics teachers, perceptions related to learning mathematics, and their mathematical resilience. The identified groups of students were labeled "cluster 1" and "cluster 2". The cluster 1 accounted for 51.7% (n = 257), and the cluster 2 account for 48.3% (n = 240) of the sample.



	MT-SP	MT-KP	MT-SA	LM-CP	LM-FP	LM-RE	MRS-V	MRS-S	MRS-G
Cluster 1	-0,71	-0,50	0,52	0,34	-0,77	-0,58	-0,64	-0,17	0,15
Cluster 2	0,66	0,46	-0,47	-0,32	0,72	0,54	0,60	0,10	-0,14

Figure 1. Student profiles

Note. MT-SP = mathematics teacher as a supportive person, MT-KP = mathematics teacher as a knowledgeable person, MT-SA = mathematics teacher as a source of anxiety, LM-CP = learning mathematics as a challenging process, LM-FP = learning mathematics as a fun process, LM-RE = learning mathematics as a process requiring effort, MRS-V= value, MRS-S=struggle, MRS-G=growth.

Figure 1 displays the z scores of the nine variables for cluster 1 and cluster 2 solutions. According to Figure 1, while the students in cluster 1 perceive a mathematics teacher as a supportive and knowledgeable person, the students in cluster 2 perceive mathematics teachers as a source of anxiety. When the two clusters were examined regarding perceptions about mathematics learning, results showed that the students in cluster 2 perceive mathematics learning as a challenging process. In contrast, cluster 1 perceives mathematics learning as a fun process requiring effort. The students in cluster 1 more values mathematics. Students in cluster 2 have a higher belief level that everyone cannot improve their mathematics ability than those in cluster 1. Both groups are similar in terms of scores obtained from the struggle subdimension of the MRS.

The Validity of Cluster Analysis Results

A series of MANOVA was conducted to examine the effects of age and gender on the research variables. The results demonstrated that the multivariate impact of both gender and age was significant o research variables. Whether the students' perceptions of mathematics teachers, perceptions of mathematics learning, and their mathematical resilience levels are significant variables to form the student profiles were tested using MANCOVA considering gender and age as covariates. Thus, gender and age were included as covariates in MANCOVA. Table 3 shows the MANCOVA results.

Table 3. MANCOVA results

	Cluster 1		Cluster 2			
	M	**S**	**M**	**S**	**F**	η_p^2
MT-supportive person	38.73	7.36	49.71	4.03	392.79***	.44
MT-knowledgeable person	19.07	3.79	22.69	2.79	133.20***	.21
MT- a source of anxiety	7.89	3.30	4.64	2.46	144.37***	.22
ML-challenging process	17.05	5.28	13.16	5.87	56.29***	.10
ML-fun process	13.71	4.42	21.91	2.82	550.50***	.52
ML-required effort	19.27	3.77	23.27	1.97	203.13***	.29
MRS-Value	29.68	6.01	36.93	2.68	278.49***	.36
MRS-Struggle	27.35	4.93	28.93	4.41	12.85***	.03
MRS-Growth	15.17	4.32	13.87	4.69	10.24**	.02

Note. M = mean, S = standard deviation, MT = mathematics teacher, ML = mathematics learning, MRS = mathematical resilience scale, **p<.01, ***p<.001, η_p^2 = partial eta square.

According to Table 3, cluster 1's perceptions regarding that a mathematics teacher is a supportive person ($F_{(1)}$= 392.79, p < .001, η_p^2 = .44) and that a mathematics teacher is a knowledgeable person ($F_{(1)}$= 133.20, p < .001, η_p^2 = .21) are significantly higher than cluster 2. In contrast, cluster 2's perceptions regarding that a mathematics teacher is a source of anxiety are significantly higher than cluster 1 ($F_{(1)}$ = 144.37, p < .001, η_p^2 = .22).

When the two clusters are examined in terms of perceptions about mathematics learning, it is seen that cluster 2's perceptions regarding that learning mathematics is a challenging process are significantly higher than cluster 1 ($F_{(1)}$= 56.29, p < .001, η_p^2 = .1). In contrast, cluster 1's perceptions regarding that learning mathematics is a fun process are significantly higher than cluster 2 ($F_{(1)}$= 550.50, p < .001, η_p^2 = .52). Cluster 1's perceptions that learning mathematics is a process requiring effort are significantly higher than cluster 2 ($F_{(1)}$= 203.13, p < .001, η_p^2 = .29).

In addition, in Table 3, it is seen that cluster 1's perception levels that mathematics is valuable ($F_{(1)}$= 278.49, p < .001, η_p^2 = .36) and perceptions of the necessity of effort in math ($F_{(1)}$= 12.85, p < .001, η_p^2 = .03) are higher than cluster 2, while cluster 2's perceptions that people cannot improve their mathematics knowledge are higher than the cluster 1 ($F_{(1)}$= 10.24, p < .01, η_p^2 = .02). Generally, cluster analysis and MANCOVA results showed that students' perceptions of mathematics teachers, perceptions related to learning mathematics, and their mathematical resilience are the significant predictors of the students' profiles. They can be used to form distinct profiles.

Comparison of the Mathematics Achievement of Two Distinct Profiles

Finally, whether students' mathematics achievement is differentiated among two distinct profiles was tested. Before the comparison, the effects of age and gender on mathematics achievement were examined. The results demonstrated that the impact of both gender and age was significant in mathematics achievement. Thus, to compare students' mathematics achievement in distinct clusters, it was decided to control gender and age. The results revealed that students' mathematics achievement in cluster 1 (M = 87.38; S = 14.27) is significantly higher than in cluster 2 (M = 81.33; S= 15.94; $F_{(1)}$= 18.06, p < .001, η_p^2 = .04).

Discussion

The primary purpose of the present study was to examine if students' perceptions of mathematics teachers, perceptions of mathematics learning, and their mathematical resilience can be used to divide students into clusters with similar profiles. Whereas the other purpose was to compare the differences in students' perceptions of mathematics teachers, perceptions of mathematics learning, their mathematical resilience, and their mathematics achievement between profiles. According to the results, two clusters were formed based on students' perceptions of mathematics teachers, perceptions of mathematics learning, and their mathematical resilience. The results showed that the students who had perceptions of mathematics teachers as a supportive and knowledgeable person also had higher mathematics achievement than those who had perceptions of mathematics teachers as a source of anxiety. These results are consistent with several research findings (e.g., Federici & Skaalvick, 2014a) and can be explained based on the negative effect of fear on students' achievement and the beneficial effects of teacher's support on students' academic achievement (Federici & Skaalvick, 2014a). For example, it is known that allowing students to make individual choices by teachers in the classroom environment positively affects students' relationships with mathematics (Bartholomew, 2007). However, recent study results showed that teachers' support when students select tasks and videos where teachers give lectures has a more significant impact on a student's relationship with mathematics and their self-regulated learning skills than students make these choices individually on their own (Samuelsson, 2021). These findings also point out that students' perceptions of mathematics teachers are a crucial variable that may lead to differences in their mathematics achievement.

Furthermore, results indicated that the students who perceived mathematics teachers as supportive and knowledgeable people also perceived mathematics learning as a fun process, compared to those who perceived mathematics teachers as the source of anxiety. Given that perceived teacher support is positively related to academic enjoyment (Sakiz et al., 2012) and its role in reducing students' burnout level (Romano et al., 2021), it could be better understood why students in cluster 1 perceive learning mathematics as a fun process and why they perceive mathematics teachers as supportive people. Another finding of the present study was that students who perceive mathematics learning as a fun process also perceive learning mathematics as a process requiring effort at the same time. Compared to students who perceive learning mathematics as a challenging process, these students have higher mathematics achievement, and they perceive mathematics as more valuable. Furthermore, these students give more value to mathematics. These results are consistent with several research findings, which stated that effort is positively related to students' achievement and utility value (Carbonaro, 2005; Chouinard et al., 2007; den Brok et al., 2005; Fisher, Waldrip, & den Brok, 2005; Sakiz et al., 2012). Given that the feeling of difficulty regarding mathematics negatively correlates with mathematics achievement (Ma, 1997; Yalçın, 2012), motivation (O'Brien & Crandall, 2003), and effort in mathematics (Hannula & Laakso, 2011; Yalçın, 2012), it can be better understood why students who perceive mathematics learning as a challenging process tend to have lower mathematics achievement. Such a finding can also explain why the students who perceive mathematics learning as a challenging process and those who perceive mathematics learning as a process requiring effort are classified into different profiles (Hannula & Laakso, 2011; Ma, 1997; Mutadi & Ngirande, 2014; Yalçın, 2012). Human behavior is mainly perception-driven (Segal, 1998), and these findings show that negative perceptions might cause low mathematics achievement.

Having the perception that mathematics is valuable is one of the most critical characteristics of mathematical resilience (Hutauruk & Priatna, 2017; Kooken et al., 2013, 2016). According to the current research results, students who perceive mathematics teachers as supportive people also believe that mathematics is crucial for future purposes. This finding supports the previous research results, which showed that teacher support is positively related to the perceptions that mathematics is useful and valuable (Federici & Skaalvick, 2014a, 2014b). Moreover, students who perceive mathematics as valuable are also aware of the necessity of effort in math. Value is an essential concept in achievement in the educational context mentioned in many theories. For example, according to the self-determination theory, understanding the personal benefit of activity supports self-determination. When the value of an action is internalized, people become willing to do it (Deci, Eghrari, Patrick, & Leone, 1994; Grolnick & Ryan, 1989) because values have a motivating aspect in terms of the power to sustain a behavior (Rheinberg, Vollmeyer, & Rollett, 2000). According to the expectancy-value theory of achievement motivation, students' expectations and value attributed to achievement are handled as significant predictors of engagement and performance (Eccles, Wigfield, & Schiefele, 1998; Pintrich & Schrauben, 1992). Indeed, relevant research results have shown that students' motivation to achieve their future goals and their perception that mathematics lessons are an essential tool (perceived instrumentality) in achieving such goals significantly explain their mathematics achievement (Eren, 2009; Husman, McCann, & Crowson, 2000; Özçetin, 2010; Başören, 2015).

Conclusion

As a summary, the results lead to three major conclusions. First, clusters formed in the present study differentiated according to students' perceptions of mathematics teachers, perceptions of learning mathematics, their mathematical resilience level, and mathematics achievement. Second, students in cluster 1 have higher mathematics achievement, more value mathematics, perceive mathematics teachers as knowledgeable and supportive people, perceive learning mathematics as a fun process requiring effort. Third, students in cluster 2 have lower mathematics achievement, perceive mathematics teachers as the source of anxiety, perceive learning mathematics as a difficult process, and low belief level that mathematics ability can be improved. From the overall results of the present study, it could be concluded that examining middle school students' perceptions of mathematics teachers, perceptions of learning mathematics, and their mathematical resilience could be highly beneficial in understanding the factors influencing students' mathematics achievement during the middle school period.

Limitations

The present study also has some limitations. First, although a sufficient sample size was used to determine clusters based on research variables, this study should be replicated in larger samples to increase the generalizability of the research results. Second, although the present study aimed to cluster students based on research variables, the results obtained from this study do not reflect the cause-result relationships between the research variables. Experimental research may be conducted to observe the causal effects of the research variables in future research. Third, the findings obtained from this research are limited to mathematics courses. Therefore, the present study results cannot be generalized to other classes. In future research, students' perceptions and resilience levels related to diverse lessons should be examined together with academic achievement.

Suggestions

The present study results have some implications for mathematics education. First, the current study results show that perceptions about mathematics teachers correlate with students' mathematics achievement. Students who perceive mathematics teachers as a supportive and knowledgeable person have higher mathematics achievement compared to those who perceive mathematics teachers as a source of anxiety. Mathematics teachers should recognize that students' perceptions regarding their teacher may negatively or positively relate to their achievement. Thus, mathematics teachers should support students and not exhibit behaviors that cause them anxiety. They may help their students reduce their perceptions that mathematics teachers are a source of anxiety during the lessons. Another critical finding obtained from this research is that the students who perceive that learning mathematics is a challenging process have lower achievement than those who perceive this as a fun process. Therefore, in addition to appearing as a support element in studying mathematics, mathematics teachers should make their lessons more fun. They should design activities that can help improve students' performance in mathematics to reduce their perceptions that mathematics learning is a challenging process. According to the results, students who find mathematics valuable have higher mathematics achievement than the others. To increase students' perception that mathematics is a valuable subject, teachers should emphasize that mathematics is an important tool that can help the students achieve their future goals.

References

Abd-El-Fattah, S. (2018). A validation study of the control-value theory within the domain of mathematics at high school: A latent profile analysis. *Journal of Psychology & Psychotherapy, 8*(5), 1-15.

Arbuckle, J. L. (2016). IBM SPSS Amos 24 user's guide. Chicago: Amos Development Corporation.

Arıkan, S., van de Vijver, F. J. R., & Yağmur, K. (2016). Factors contributing to mathematics achievement differences of Turkish and Australian students in TIMSS 2007 and 2011. *Eurasia Journal of Mathematics, Science and Technology Education, 12*(8), 2039-2059.

Bartholomew, B. (2007). Why can't we always get what we want?. *Phi Delta Kappan 88*(8), 593-598.

Başören, M. (2015). *The relation of high school students' engagement in maths class with the variables of emotion regulation, perceived instrumentality, hope, and maths achievement* (Unpublished doctoral dissertation). Bolu Abant Izzet Baysal University, Bolu.

Bawuah, B., Sare, Y. A., & Kumah, S. P. (2014). What perceived practices of teachers contribute to students' mathematics learning and achievement at the SHS level? A case study in the central region of Ghana. *Journal of Education and Practice, 5*(8), 139-148.

Bergman, L. (1998). A pattern-oriented approach to studying individual development: Snapshots and processes. In R. Cairns, L. Bergman, & J. Kagan (Eds.), *Methods and models for studying the individual* (pp. 83-121). New York: Russell Sage.

Borman, G. D., & Overman, L. T. (2004). Academic resilience in mathematics among poor and minority students. *The Elementary School Journal, 104*(3), 177-195.

Carbonaro, W. (2005). Tracking, students' effort, and academic achievement. *Sociology of Education, 78*(1), 27-49.

Chouinard, R., Karsenti, T., & Roy, N. (2007). Relations among competence beliefs, utility value, achievement goals, and effort in mathematics. *British Journal of Educational Psychology, 77*(3), 501-517.

Cropp, I. (2017). Using peer mentoring to reduce mathematical anxiety. *Research Papers in Education, 32*(4), 481-500.

Çağlayan, Ç. (2018). *Matematik dersine ilişkin metaforlar ve matematiksel yılmazlık: Bir kümeleme analizi yaklaşımı* (Unpublished master's thesis). Bolu Abant Izzet Baysal University, Bolu.

Daher, W., Anabousy, A., & Jabarin, R. (2018). Metacognition, positioning, and emotions in mathematical activities. *International Journal of Research in Education and Science, 4*(1), 292-303.

Deci, E. L., Eghrari, H., Patrick, B. C., & Leone, D. R. (1994). Facilitating internalization: The self determination theory perspective. *Journal of Personality, 62*(1), 119-142.

den Brok, P., Fisher, D., & Scott, R. (2005). The importance of teacher interpersonal behavior for student attitudes in Brunei's primary science classes. *International Journal of Science Education, 27*(7), 765-779.

den Brok, P., van Tartwijk, J., Wubbels, T., &Veldman, I. (2010). The differential effect of the teacher student interpersonal relationship on student outcomes for students with different ethnic backgrounds. *British Journal of Educational Psychology, 80*, 199-221.

Eccles, J. S., Wigfield, A., & Schiefele, U. (1998). Motivation to succeed. In W. Damon & N. Eisenberg (Eds.), *Handbook of child psychology: Social, emotional, and personality development* (pp. 1017-1095). Hoboken, NJ, US: John Wiley & Sons Inc.

Eren, A. (2009). Exploring the effects of changes in future time perspective and perceived instrumentality on graded performance. *Electronic Journal of Research in Educational Psychology, 7*(3), 1217-1248.

Federici, R. A., & Skaalvik, E. M. (2014a). Students' perceptions of emotional and instrumental teacher support: Relations with motivational and emotional responses. *International Education Studies, 7*(1), 21-36.

Federici, R. A., & Skaalvik, E. M. (2014b). Students' perception of instrumental support and effort in mathematics: The mediating role of subjective task values. *Social Psychology of Education, 17*(3), 527-540.

Fisher, D., Waldrip, B., & den Brok, P. (2005). Students' perceptions of primary teachers' interpersonal behavior and cultural dimensions in the classroom environment. *International Journal of Educational Research, 43*(1/2), 25-38.

Goetz, T., Frenzel, A. C., Hall, N. C., & Pekrun, R. (2008). Antecedents of academic emotions: Testing the internal/external frame of a reference model for academic enjoyment. *Contemporary Educational Psychology, 39*(1), 9-33.

Grolnick, W. S., & Ryan, R. M. (1989). Parent styles associated with children's self-regulation and competence in school. *Journal of Educational Psychology, 81*(2), 143-154.

Gürefe, N., & Akçakın, V. (2018). The Turkish adaptation of the mathematical resilience scale: Validity and reliability study. *Journal of Education and Training Studies, 6*(4), 38-47.

Hanin, V., & Van Nieuwenhoven, C. (2019). Emotional and motivational relationship of elementary students to mathematical problem-solving: A person-centered approach. *European Journal of Psychology of Education, 34*(4), 705-730.

Hannula, M. S., & Laakso, J. (2011). *The structure of mathematics-related perceptions, attitudes, and motivation among Finnish grade 4 and grade 8 students.* Proceedings of the 35th Conference of the International Group for the Psychology of Mathematics Education, Ankara.

Helm, F., Arens, A. K., & Möller, J. (2020). Perceived teacher unfairness and student motivation in math and German: An application of the generalized internal/external frame of the reference model. *Learning and Individual Differences, 81,* 1-10.

Husman, J., McCann, E. J., & Crowson, H. M. (2000). Volitional strategies and future time perspective: Embracing the complexity of dynamic interactions. *International Journal of Educational Research, 33*(7-8), 777-799.

Hutauruk, A. J. B., & Priatna, N. (2017). Mathematical resilience of mathematics education students. *Journal of Physics: Conference Series, 895*, 1-6.

Jang, H., Reeve, J., & Deci, E. L. (2010). Engaging students in learning activities: It's not autonomy support or structure, but autonomy support and structure. *Journal of Educational Psychology, 102*(3), 588-600.

Johnston-Wilder, S., & Lee, C. (2010). *Developing mathematical resilience*. Paper presented at BERA Annual Conference, Coventry, United Kingdom.

Johnston-Wilder, S., Brindley, J., & Dent, P. (2014). *A survey of mathematics anxiety and mathematical resilience among existing apprentices.* London: Gatsby Charitable Foundation.

Johnston-Wilder, S., Lee, C., Brindley, J., & Garton, E. (2015). *Developing peer coaching for mathematical resilience in post-16 students who are encountering mathematics in other subjects.* In 8th International Conference of Education, Research and Innovation, Seville, Spain.

Kooken, J., Welsh, M. E., Mccoach, D. B., Johnson-Wilder, S., & Lee, C. (2013). *Measuring mathematical resilience: an application of the construct of resilience to the study of mathematics.* American Educational Research Association (AERA) 2013 Annual Meeting: Education and Poverty: Theory, Research, Policy and Praxis, The Open University Press, San Francisco, CA.

Kooken, J., Welsh, M. E., Mccoach, D. B., Johnston-Wilder, S., & Lee, C. (2016). Development and validation of the mathematical resilience scale. *Measurement and Evaluation in Counseling and Development, 49*(3), 217-242.

Laursen, B., & Hoff, E. (2006). Person-centered and variable-centered approaches to longitudinal data. *Merrill-Palmer Quarterly, 52*(3), 377-389.

Lazarides, R., Dietrich, J., & Taskinen, P. H. (2019). Stability and change in students' motivational profiles in mathematics classrooms: The role of perceived teaching. *Teaching and Teacher Education, 79*, 164-175.

Litalien, D., Gillet, N., Gagne, M., Ratelle, C., & Morin, A. (2019). Self-determined motivation profiles among undergraduate students: A robust test of profile similarity as a function of gender and age. *Learning and Individual Differences, 70*, 39-52.

Ma, X. (1997). Reciprocal relationships between attitude toward mathematics and achievement in mathematics. *The Journal of Educational Research, 90*(4), 221-229.

Mammarella, I. C., Donolato, E., Caviola, S., & Giofrè, D. (2018). Anxiety profiles and protective factors: A latent profile analysis in children. *Personality & Individual Differences, 124*(1), 201-208.

Mutadi, P., & Ngirande, H. (2014). The influence of students' perceptions on mathematics performance. A case of a selected high school in South Africa. *Mediterranean Journal of Social Sciences, 5*(3), 431-445.

Muthen, B., & Muthen, L. (2000). Integrating person-centered and variable-centered analysis: Growth mixture modeling with latent trajectory classes. *Alcoholism: Clinical and Experimental Research, 24*(6), 882-891.

O'Brien, L. T., & Crandall, C. S. (2003). Stereotype threat and arousal: Effects on women's math performance. *Personality and Social Psychology Bulletin, 29*(6), 782-789.

Özçetin, N. (2010). *Investigating the effects of future time perspective and perceived instrumentality of vocational high school students on their graded performance and attitudes towards English* (Unpublished master's thesis). Bolu Abant Izzet Baysal University, Bolu.

Pekrun, R. (2000). Social-cognitive, control-value theory of achievement emotions. In J. Heckhausen (Ed.), *Motivational psychology of human development* (pp. 143-163). Oxford, England: Elsevier Science.

Pekrun, R. (2006). The control-value theory of achievement emotions: Assumptions, corollaries, and implications for educational research and practice. *Educational Psychology Review 18*(4), 315-341.

Pekrun, R., Elliot, A. J., & Maier, M. A. (2006). Achievement goals and discrete achievement emotions: A theoretical model and prospective test. *Journal of Educational Psychology, 98*(3), 583-597.

Pekrun, R., Goetz, T., Titz, W., & Perry, R. P. (2002). Academic emotions in students' self-regulated learning and achievement: A program of qualitative and quantitative research. *Educational Psychologist, 37*(2), 91-105.

Pintrich, P. R., & Schrauben, B. (1992). Students' motivational beliefs and their cognitive engagement in classroom academic tasks. In D. H. Schunk & J. L. Meece (Eds.), *Student perceptions in the classroom* (pp. 149-183). Hillsdale, NJ, US: Lawrence Erlbaum Associates, Inc.

Putwain, D. W., & Symes, W. (2011) Teachers' use of fear appeals in the mathematics classroom: Worrying or motivating students?. *British Journal of Educational Psychology, 81*(3), 456-474.

Reddy, R., Rhodes, J. E., & Mulhall, P. (2003). The influence of teacher support on student adjustment in the middle school years: A latent growth curve study. *Development and Psychopathology, 15*(1), 119-138.

Rheinberg, F., Vollmeyer, R., & Rollett, W. (2000). Motivation and action in self-regulated learning. In M. Boekaerts, P. R. Pintrich, & M. H. Zeidner (Eds.), *Handbook of self-regulation* (pp. 503-529). San Diego, CA: Academic Press.

Romano, L., Angelini, G., Consiglio, P., & Fiorilli, C. (2021). The effect of students' perception of teachers' emotional support on school burnout dimensions: Longitudinal findings. *International Journal of Environmental Research and Public Health, 18*(4), 1-7.

Sakiz, G., Pape, S. J., & Woolfolk-Hoy, A. (2012). Does perceived teacher affective support matter for middle school students in mathematics classrooms?. *Journal of School Psychology, 50*(2), 235-255.

Samuelsson, J. (2021). Developing students' relationships with mathematics. *Educational Action Research,* 1-15.

Satish, S. M., & Bharadhwaj, S., (2010). Information search behavior among new car buyers: A two-step cluster analysis. *IIMB Management Review, 22*(1-2), 5-15.

Segal, S. (1998). The role of contingency and tension in the relationship between theory and practice in the classroom. *Journal of Curriculum Studies, 30*(2), 199-206.

Thijs, J., & Fleischmann, F. (2015). Student-teacher relationships and achievement goal orientations: Examining student perceptions in an ethnically diverse sample. *Learning and Individual Differences, 42*, 53-63.

Umarji, O., McPartlan, P., & Eccles, J. (2018). Patterns of math and English self-concepts as motivation for college major selection. *Contemporary Educational Psychology, 53*, 146-158.

Yalçın, M. O. (2012). *The relationship between high school students' metaphors, attitudes, and achievement levels related to mathematics lessons* (Unpublished master's thesis). Bolu Abant Izzet Baysal University, Bolu.

Zee, M., & Roorda, D. L. (2018). Student-teacher relationships in elementary school: The unique role of shyness, anxiety, and emotional problems. *Learning and Individual Differences, 67*, 156-166.